

Homework Assignment #1

Due: September 17, 2015 at 4:00 p.m.

1. Let $n \geq 1$. Suppose we have a collection of $N = 2^n$ computers. They are connected into a hypercube network as follows. Each computer is labelled by a binary string of length n . No two computers have the same label. There is a wire connecting two computers if and only if the two labels of the computers differ in exactly one bit. For example, if $n = 8$ there would be a direct connection between the computers labelled 01001001 and 01011001 because the two labels differ only in the value of their fourth bits. When $n = 3$, the network forms a cube as shown in the diagram below.

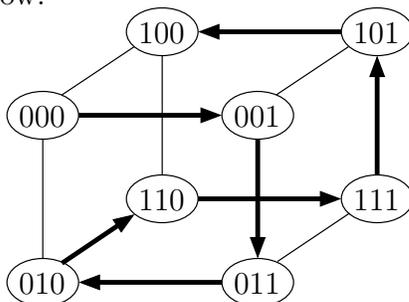
Suppose we want the computer with label 000...0 to send some information to all other computers, but we only want to transmit messages across $N - 1$ of the edges. Willemina figured out how to do this and wrote a recursive algorithm to print out the order in which the information should be passed around the network.

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1 LIST( $n$ ) : list of strings
2   if  $n$  is 1 then output  $\langle 0, 1 \rangle$ 
3   else
4      $L \leftarrow \text{LIST}(n - 1)$ 
5     let  $x_1, x_2, \dots, x_k$  be the elements of  $L$  (in order)
6     output  $\langle 0x_1, 0x_2, \dots, 0x_{k-1}, 0x_k, 1x_k, 1x_{k-1}, \dots, 1x_2, 1x_1 \rangle$ 
7   end if
8 end LIST

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For example, LIST(3) outputs $\langle 000, 001, 011, 010, 110, 111, 101, 100 \rangle$. Thus, according to Willemina's scheme, the message would be sent around the network as shown by the arrows below.



Prove that for all $n \geq 1$, the output of LIST(n) is correct. In other words, prove that it has the following two properties.

- Every binary string of length n is printed exactly once.
- If two labels appear next to each other in the list, then there is a wire between the two computers with those labels.