EECS-1019C: ASSIGNMENT #7

Out of 20 points.

Section 3.3 [20pt]

8. [5pt] Given a real number x and a positive integer k, determine the number of multiplications used to find x^{2^k} starting with x and successively squaring (to find x^2, x^4 , and so on). Is this a more efficient way to find x^{2^k} than by multiplying x by itself the appropriate number of times?

> *By successively squaring k times, we get x^{2^k}. Thus, we can compute x^{2^k} in k multiplication steps. If we do this multiplying by 2 each time, this would require $2^k - 1$ multiplications. So yes, it is much more efficient!*

14. [10pt] There is a more efficient algorithm (in terms of the number of multiplications and additions used) for evaluating polynomials than the conventional algorithm described in the previous exercise. It is called Horners method. This pseudocode shows how to use this method to find the value of $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ at $x = c$.

```
1 procedure Horner(c, a_0, a_1, a_2, ..., a_n: real numbers)
2     y := a_n
3     for i := 1 to n
4         y := y * c + a_{n-i}
5     return y {a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0}
```

a. [5pt] Evaluate $3x^2 + x + 1$ at $x = 2$ by working through each step of the algorithm showing the values assigned at each assignment step.

> $y_0 = 3$
> $y_1 = 7$
> $y_2 = 15$
> *We return y_2.*

b. [5pt] Exactly how many multiplications and additions are used by this algorithm to evaluate a polynomial of degree n at $x = c$? (Do not count additions used to increment the loop variable.)

> *n multiplications and n additions. One each in step (4) which we do n times in the for-loop.*

18. [5pt] How much time does an algorithm take to solve a problem of size n if this algorithm uses $2n^2 + 2^n$ operations, each requiring 10^{-9} seconds, with these values of n?

a. 10

$\sim 1.224 \times 10^{-6}$ *seconds*

b. 20

$\sim 1.05 \times 10^{-3}$ *seconds*

c. 50

$\sim 1.13 \times 10^{6}$ *seconds, about* 13 *days*

d. 100

$\sim 1.27 \times 10^{21}$ *seconds, about* 4×10^{13} *years*