EECS-1019C: Assignment #5

Out of 30 points.

Section 2.4 [18pt]

10. [10pt] Find the first six terms of the sequence defined by each of these recurrence relations and initial conditions.

a. [2pt] $a_n = -2a_{n-1}$, $a_0 = -1$

$a_1, \ldots, a_6 = 2, -4, 8, -16, 32, -64$

b. [2pt] $a_n = a_{n-1} - a_{n-2}$, $a_0 = 2$, $a_1 = -1$

$a_2, \ldots, a_7 = -3, -2, 1, 3, 2, -1, -3$

c. [2pt] $a_n = 3a_{n-1}^2$, $a_0 = 1$

$a_1, \ldots, a_6 = 3, 27, 2187, 14348907, 617673396283947, 1144561273430837494885949696427$

d. [2pt] $a_n = na_{n-1} + a_{n-2}^2$, $a_0 = -1$, $a_1 = 0$

$a_2, \ldots, a_7 = 1, 3, 13, 74, 613, 9767$

e. [2pt] $a_n = a_{n-1} - a_{n-2} + a_{n-3}$, $a_0 = 1$, $a_1 = 1$, $a_2 = 2$

$a_3, \ldots, a_8 = 2, 1, 1, 2, 2, 1$

30. [8pt] What are the values of these sums, where $S = \{1, 3, 5, 7\}$?

a. [2pt] $\sum_{j \in S} j$

16

b. [2pt] $\sum_{j \in S} j^2$

84

c. [2pt] $\sum_{j \in S} 1/j$

176/105

d. [2pt] $\sum_{j \in S} 1$

4

Section 2.5 [12pt]

2. [12pt] Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

a. [2pt] the integers greater than 10

$\{x \mid x \in \mathbb{Z} \wedge x > 10$ *Countably infinite:* $x \leftrightarrow x - 10$

b. [2pt] the odd negative integers

$\{x \mid \exists y \in \mathbb{N}(x = -(2y + 1)\}$ *is countably infinite:* $x \leftrightarrow -(x + 1)/2 + 1$

c. [2pt] the integers with absolute value less than 1,000,000

finite: there are 1,999,999 of them

d. [2pt] the real numbers between 0 and 2

uncountably infinite

e. [2pt] the set $A \times \mathbb{Z}^+$ where $A = \{2, 3\}$

$\{(x, y) \mid x \in \{2, 3\} \wedge y \in \mathbb{Z}^+\}$ *is countably infinite:* $(2, y) \leftrightarrow 2y - 1$ *and* $(3, y) \leftrightarrow 2y$

f. [2pt] the integers that are multiples of 10

$10x \mid x \in \mathbb{Z}$ *is countably infinite:* $x \leftrightarrow 10x$