EECS-1019c: Assignment #1

Out of N points.

Section 1.1 [20pt] (Choose 10 of 20)

2. [10pt] Which of these are propositions? What are the truth values of those that are propositions?

a. [2pt] Do not pass go.

Not a proposition. (It is an imperative.)

b. [2pt] What time is it?

Not a proposition. (It is a question.)

c. [2pt] There are no black flies in Maine.

Is a proposition. False, *according to the textbook. And it is factually false too.*

d. [2pt] $4 + x = 5$.

Not a proposition. "x" is a variable.

e. [2pt] The moon is made of green cheese.

Is a proposition. False.

f. [2pt] $2^n \geq 100$.

Not a proposition. "n" is a variable.

12. [10pt] Let p, q, and r be the propositions.

p: You have the flu.

q: You miss the final examination.

r: You pass the course.

a. [2pt] $p \rightarrow q$

If you have the flu, then you miss the final exam.

b. [2pt] $\neg q \leftrightarrow r$

You pass the course if and only if you do not miss the final exam.

c. [2pt] $q \rightarrow \neg r$

If you miss the final exam, then you do not pass the course.

d. [2pt] $p \vee q \vee r$

You have the flu, miss the final exam, or pass the course.

e. [2pt] $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

If you have the flu or miss the final exam, then you do not pass the course.

f. [2pt] $(p \wedge q) \vee (\neg q \wedge r)$

You have the flu and miss the final exam, or you do not miss the final exam and pass the course.

24. [10pt] Write each of these statements in the form "if p, then q" in English.

[Hint: Refer to the list of common ways to express conditional statements provided in this section.]

a. [2pt] I will remember to send you the address only if you send me an e-mail message.

> *If you send me an e-mail message, then I will remember to send you the address.*

b. [2pt] To be a citizen of this country, it is sufficient that you were born in the United States.

> *If you were born in the United States, then you can be a citizen of the United States.*

c. [2pt] If you keep your textbook, it will be a useful reference in your future courses.

> *If you keep your textbook, then it will be a useful reference in your future courses.*

d. [2pt] The Red Wings will win the Stanley Cup if their goalie plays well.

> *If the Red Wings's goalie plays well, they will win the Stanley Cup.*

e. [2pt] That you get the job implies that you had the best credentials.

> *If you get the job, then you had the best credentials.*

f. [2pt] The beach erodes whenever there is a storm.

> *If there is a storm, then beach erodes.*

g. [2pt] It is necessary to have a valid password to log on to the server.

> *If you can log on to the server, then you must have a valid password.*

h. [2pt] You will reach the summit unless you begin your climb too late.

> *If you begin your climb too late, then you will not reach the summit.*

Section 1.2 [10pt]

10. [5pt] Are these system specifications consistent? "Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded."

> u: *Software is being upgraded.*
>
> a: *Users can access the file system.*
>
> f: *Users can save files.*
>
> *The three specifications then are*
>
> 1. $u \to \neg a$
> 2. $a \to f$
> 3. $\neg f \to \neg u$
>
> *The specifications are odd, but they are consistent. To see this, consider u is* true *(the system is being upgraded), a is* false *(users cannot access the file system), and f is* true *(users cannot access the file system). Specification (1) is* true *as u and $\neg a$ are* true. *Specification (2) is* true *as a is* false. *Specification (3) is* true *as $\neg f$ is* false.
> *This seems odd as we might think, "How can the user save files if he or she cannot access the file system@?" But nothing in the specifications states this.*

16. [5pt] An explorer is captured by a group of cannibals. There are two types of cannibals—those who always tell the truth and those who always lie. The cannibals will barbecue the explorer unless he can determine whether a particular cannibal always lies or always tells the truth. He is allowed to ask the cannibal exactly one question.

a. Explain why the question "Are you a liar?" does not work.

> *If he is a liar (that is, always lies), he will answer, "Yes." If he is honest (that is, always tells the truth), he will answer, "Yes." Thus, his answer does not help the explorer to know whether he is honest or is a liar.*

b. Find a question that the explorer can use to determine whether the cannibal always lies or always tells the truth.

> *"If I cannot determine whether you are honest or are a liar, will you barbecue me?"*
> *If the cannibal is a liar, he must say, "No." Otherwise, he would be telling the truth, which he cannot do.*
> *If the cannibal is honest, he must say, "Yes." Otherwise, he would be lying, which he cannot do.*

Section 1.3 [20pt] (Choose 10 of 13)

6. [2pt] Use a truth table to verify the first De Morgan law $\neg(p \vee q) \equiv \neg p \vee \neg q$.

p	q	$\neg(p \vee q)$	$\neg p \vee \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

8. [8pt] Use De Morgans laws to find the negation of each of the following statements.

a. [2pt] Kwame will take a job in industry or go to graduate school.

Kwame will not take a job in industry, nor will he go to graduate school.

b. [2pt] Yoshiko knows Java and calculus.

Yoshiko does not know Java or she does not know calculus.

c. [2pt] James is young and strong.

James is not young or he is not strong.

d. [2pt] Rita will move to Oregon or Washington.

Rita will move neither to Oregon nor Washington.

10. [8pt] Show that each of these conditional statements is a tautology by using truth tables.

a. [2pt] $[\neg p \wedge (p \vee q)] \rightarrow q$

p	q	$p \vee q$	$[\neg p \wedge (p \vee q)]$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

b. [2pt] $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

p	q	r	$(p \rightarrow q)$	$(q \rightarrow r)$	$(p \rightarrow q)$ $\wedge(q \rightarrow r)$	$(p \rightarrow r)$	$[(p \rightarrow q) \wedge (q \rightarrow r)]$ $\rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

c. [2pt] $[p \wedge (p \rightarrow q)] \rightarrow q$

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

d. [2pt] $[(p \vee q) \wedge (p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow r$

p	q	r	$p \vee q$	$p \rightarrow q$	$q \rightarrow r$	$(p \vee q)$ $\wedge(p \rightarrow q)$ $\wedge(q \rightarrow r)$	$[(p \vee q)$ $\wedge(p \rightarrow q)$ $\wedge(q \rightarrow r)] \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	T
T	F	T	T	F	T	F	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

30. [2pt] Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

$(p \vee q) \wedge (\neg p \vee r)$

1	$\equiv$	$(\neg p \rightarrow q) \wedge (p \rightarrow r)$	*[implication equiv.]*
2	$\equiv$	$T \wedge ((\neg p \rightarrow q) \wedge (p \rightarrow r))$	*[identity]*
3	$\equiv$	$(\neg p \vee p) \wedge ((\neg p \rightarrow q) \wedge (p \rightarrow r))$	*[negation]*
4	$\equiv$	$(\neg p \wedge (\neg p \rightarrow q) \wedge (p \rightarrow r))$ $\vee\ (p \wedge (\neg p \rightarrow q) \wedge (p \rightarrow r))$	*[distributive]*
5	$\rightarrow$	$(q \wedge F) \vee (F \wedge r)$	*[modus ponens]*
6	$\equiv$	$(q \vee r)$	*[domination]*

34. [6pt] Find the dual of each of these compound propositions.

a. [2pt] $p \vee \neg q$

$p \wedge \neg q$

b. [2pt] $p \wedge (q \vee (r \wedge T))$

$p \vee (q \wedge (r \vee F))$

c. [2pt] $(p \wedge \neg q) \vee (q \wedge F)$

$(p \vee \neg q) \wedge (q \vee T)$