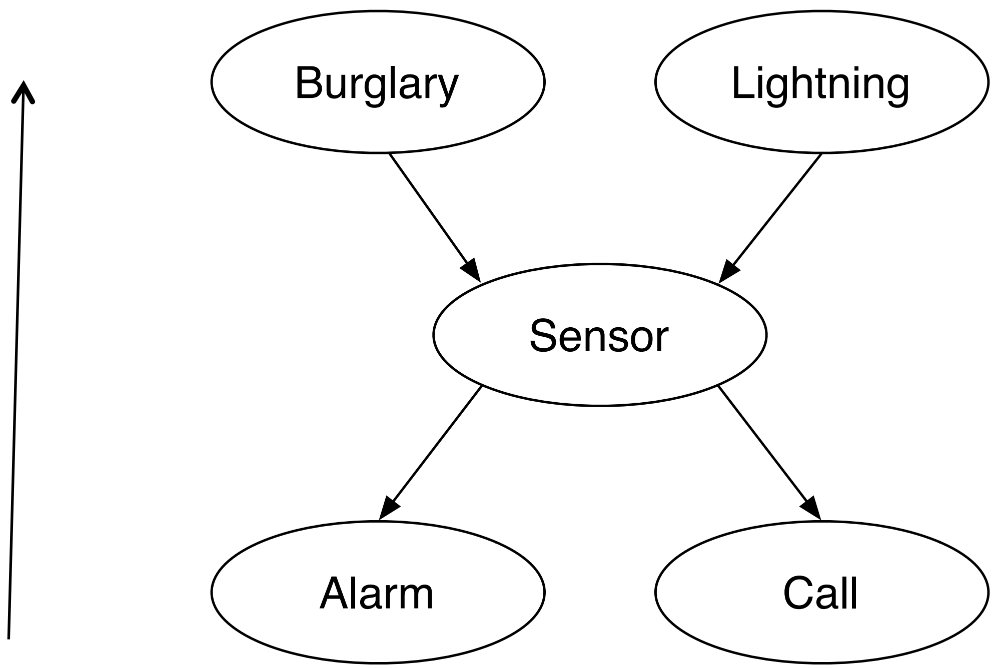
Bayesian Networks
Part 2 of 4
Defining probability equations

Compute P(alarm)

◊ The computation uses backward chained reasoning

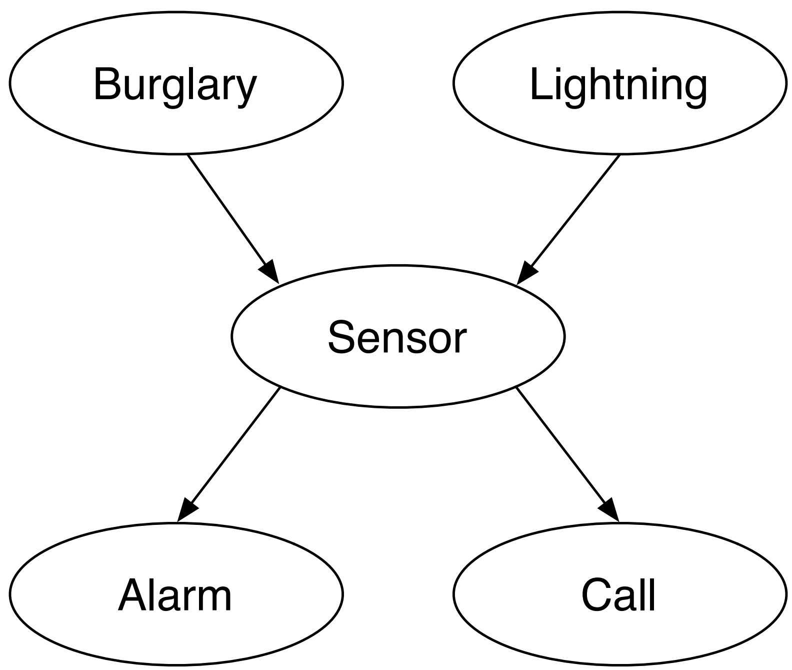
» **Recall we have the probability tables for every event**



Compute P(alarm) – 2

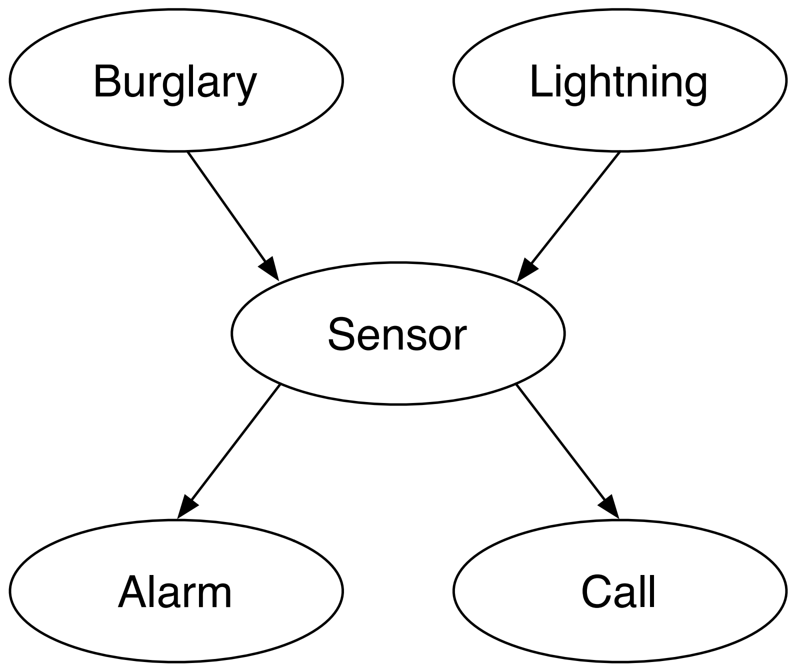
◊ The computation uses backward chained reasoning

» **The probability computations (arithmetic) are in the forward direction**



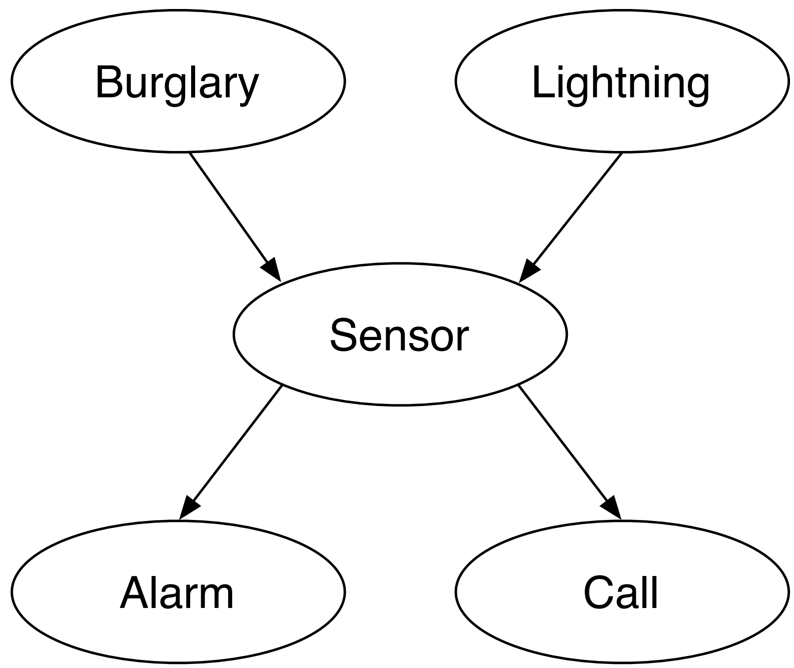
Compute P(alarm) – 3

◊ The computation uses backward chained reasoning
  » **The probability computations (arithmetic) are in the forward direction**
    > **From known values to unknown values.**



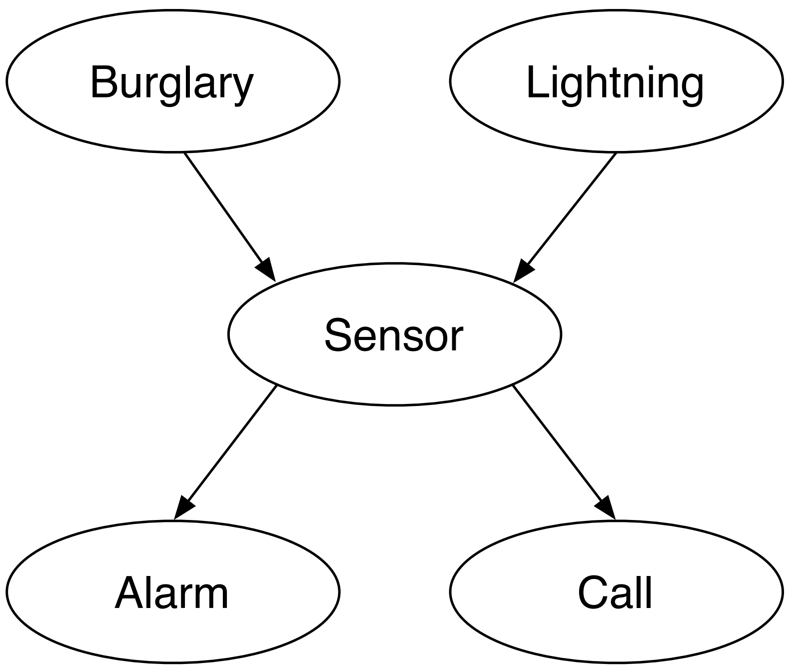
P(alarm) equation

» **What is the probability equation for P(alarm)?**



P(alarm) equation – 2

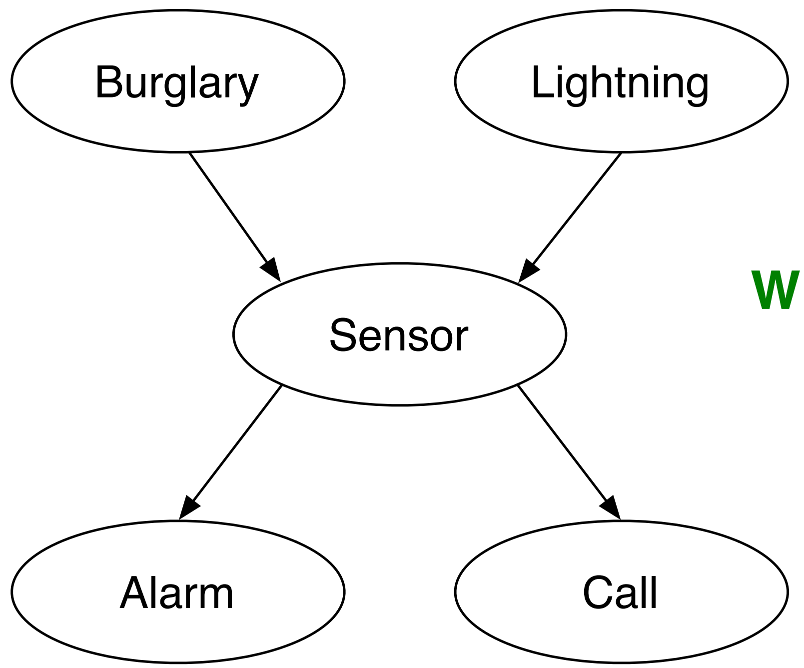
- The probability equation
  - P(alarm) = P(alarm | sensor) * P(sensor)
    + P(alarm | ~ sensor) * P(~ sensor)



P(alarm) equation – 3

◊ The probability equation

» **P(alarm) = P(alarm | sensor) * P(sensor) + P(alarm | ~ sensor) * P(~ sensor)**



What do we do?

P(alarm) evaluation

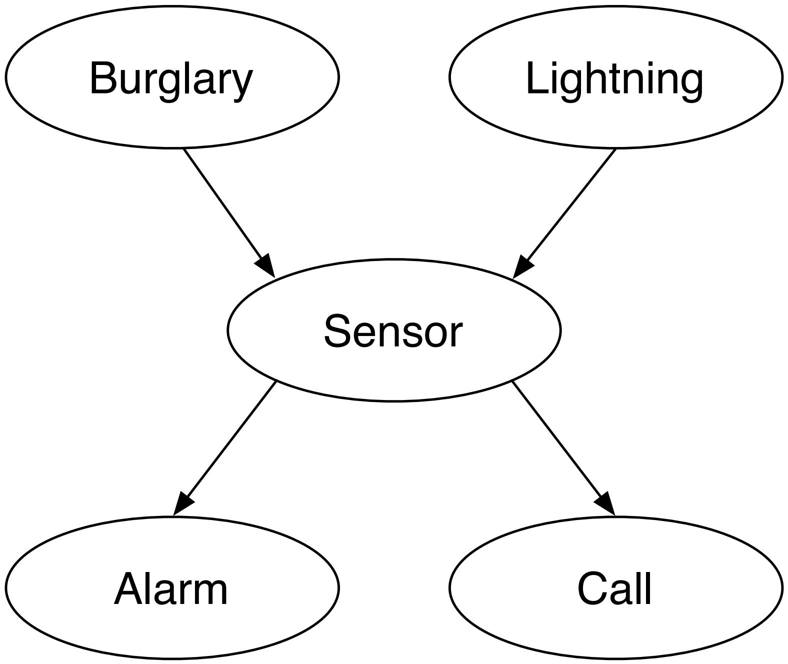
◊ The probability equation

» P(alarm) = P(alarm | sensor) * P(sensor)
    + P(alarm | ~ sensor) * P(~ sensor)

> Evaluate the right hand side

P(alarm) evaluation – 2

- The probability equation
  - P(alarm) = P(alarm | sensor) * P(sensor)
    + P(alarm | ~ sensor) * P(~ sensor)
    - Evaluate the right hand side
  - What do we know?
    What are we given?



Burglar probability tables

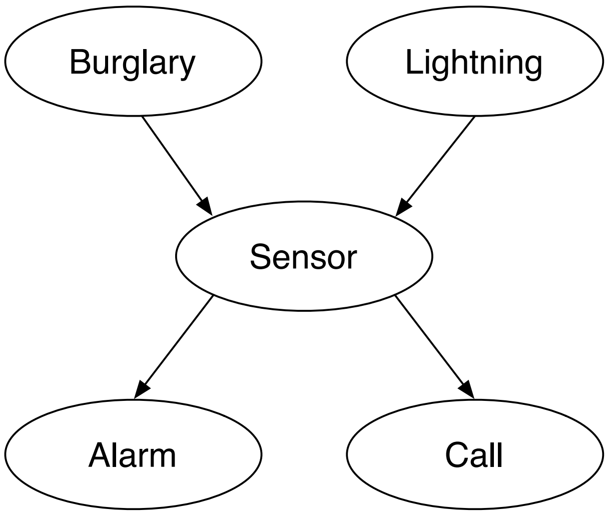
» **What do we know?**
What are we given?

◊ P(burglary) = 0.001
P(lightning) = 0.02

P(sensor | burglary, lightning) = 0.9
P(sensor | burglary, ~ lightning) = 0.9
P(sensor | ~ burglary, lightning) = 0.1
P(sensor | ~ burglary, ~ lightning) = 0.001

P(alarm | sensor) = 0.95
P(alarm | ~ sensor) = 0.001

P(call | sensor) = 0.9
P(call | ~ sensor) = 0.0



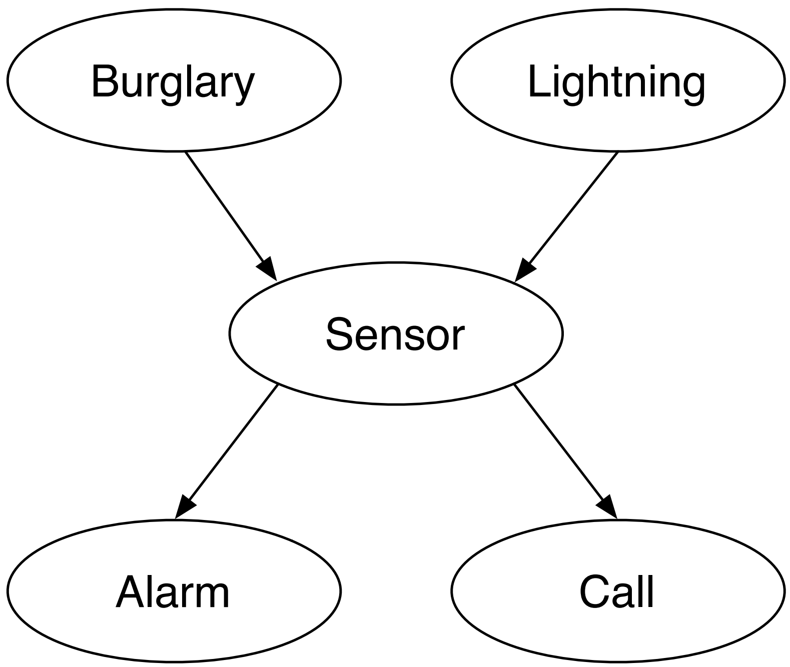
P(alarm) evaluation – 3

◊ The probability equation

» **P(alarm) = P(alarm | sensor) * P(sensor)
+ P(alarm | ~ sensor) * P(~ sensor)**

> **Evaluate the right hand side**

» **Use what we are given**



P(alarm) evaluation – 4

- The probability equation
  - **P(alarm) = P(alarm | sensor) * P(sensor) + P(alarm | ~ sensor) * P(~ sensor)**
    - **Evaluate the right hand side**
  - **P(alarm | sensor) = 0.95** **given**

P(alarm) evaluation – 5

- The probability equation
  - » P(alarm) = P(alarm | sensor) * P(sensor)
    + P(alarm | ~ sensor) * P(~ sensor)
    - > Evaluate the right hand side
  - » P(alarm | sensor) = 0.95 given
  - » P(alarm | ~ sensor) = 0.001 given

P(alarm) evaluation – 6

◊ The probability equation

» P(alarm) = P(alarm | sensor) * P(sensor)
  + P(alarm | ~ sensor) * P(~ sensor)

> Evaluate the right hand side

» P(alarm | sensor) = 0.95 given

» P(alarm | ~ sensor) = 0.001 given

» P(~ sensor) = 1 – P(sensor) probability rule

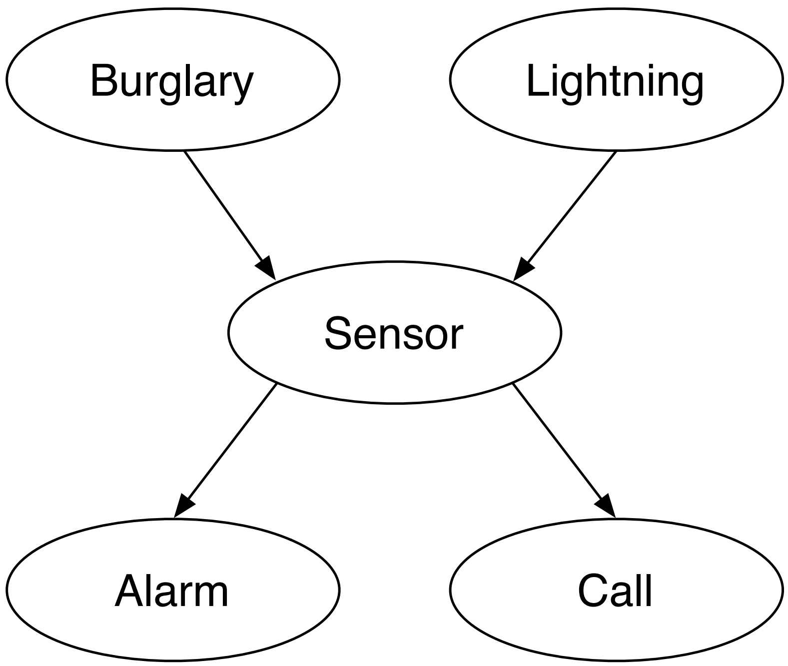
> What about P(sensor)?

P(alarm) evaluation – 7

- The probability equation
  - » P(alarm) = P(alarm | sensor) * P(sensor)
    + P(alarm | ~ sensor) * P(~ sensor)
    - > Evaluate the right hand side
  - » P(alarm | sensor) = 0.95 — given
  - » P(alarm | ~ sensor) = 0.001 — given
  - » P(~ sensor) = 1 – P(sensor) — probability rule
    - > What about P(sensor)?
      - – Need to compute it

P(sensor) equation

» **What is the probability equation for P(sensor) ?**



P(sensor) equation – 2

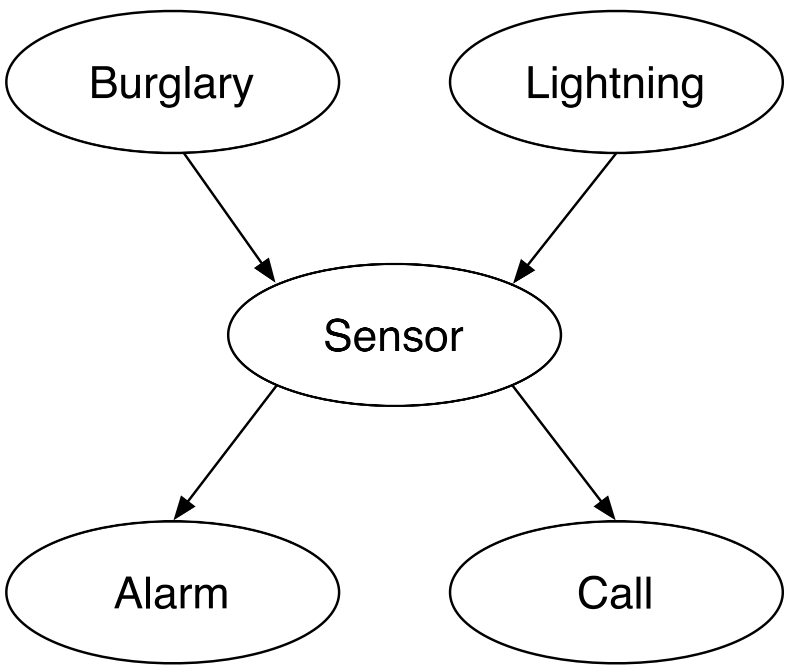
◊ The probability equation

» P(sensor) = P(sensor | burglary ^ lightning)
  * P(burglary ^ lightning)

  + P(sensor | burglary ^ ~ lightning)
  * P(burglary ^ ~ lightning)

  + P(sensor | ~ burglary ^ lightning)
  * P(~ burglary ^ lightning)

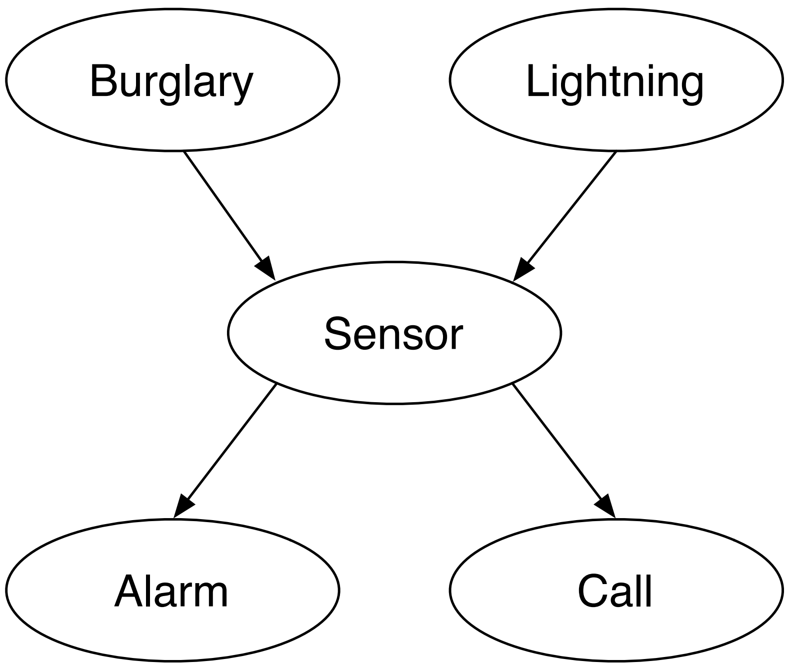
  + P(sensor | ~ burglary ^ ~ lightning)
  * P(~ burglary ^ ~ lightning)



P(sensor) equation – 3

◊ The probability equation

» P(sensor) = P(sensor | burglary ^ lightning)
  * P(burglary ^ lightning)



+ P(sensor | burglary ^ ~ lightning)
  * P(burglary ^ ~ lightning)

+ P(sensor | ~ burglary ^ lightning)
  * P(~ burglary ^ lightning)

+ P(sensor | ~ burglary ^ ~ lightning)
  * P(~ burglary ^ ~ lightning)

What do we do?

P(sensor) equation – 4

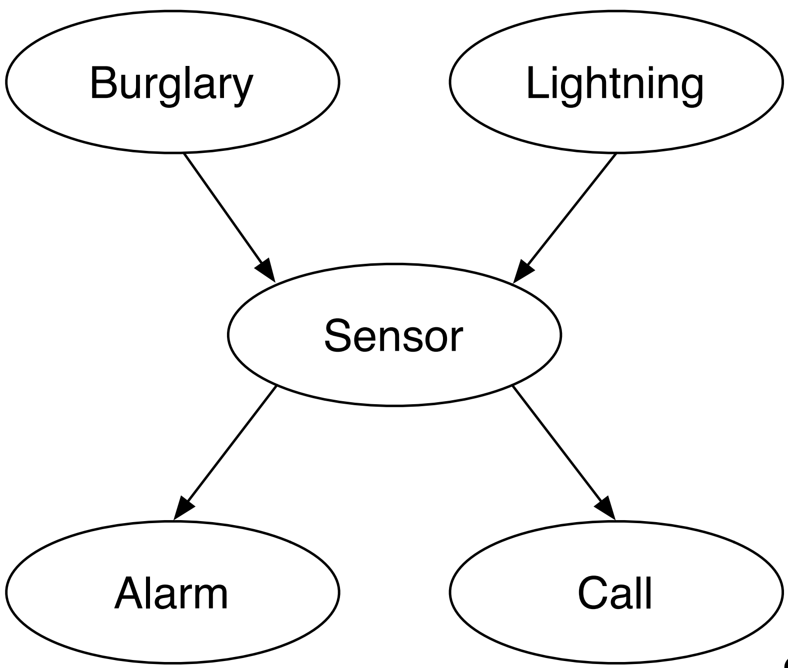
◊ The probability equation

» P(sensor) = P(sensor | burglary ^ lightning)
  * P(burglary ^ lightning)

+ P(sensor | burglary ^ ~ lightning)
  * P(burglary ^ ~ lightning)

+ P(sensor | ~ burglary ^ lightning)
  * P(~ burglary ^ lightning)

+ P(sensor | ~ burglary ^ ~ lightning)
  * P(~ burglary ^ ~ lightning)



What do we do?

– Simplify the equation
– How?

P(sensor) equation – 6

- Burglary and lightning are independent
  - > **We get the following**
    - » **P(sensor) = P(sensor | burglary ^ lightning) * P(burglary) * P(lightning)**

      + P(sensor | burglary ^ ~ lightning) * P(burglary) * P(~ lightning)

      + P(sensor | ~ burglary ^ lightning) * P(~ burglary) * P(lightning)

      + P(sensor | ~ burglary ^ ~ lightning) * P(~ burglary) * P(~ lightning)

P(sensor) equation – 7

- ◊ Burglary and lightning are independent
  - > **We get the following**
    - » **P(sensor) = P(sensor | burglary ^ lightning) * P(burglary) * P(lightning)**

      + P(sensor | burglary ^ ~ lightning) * P(burglary) * P(~ lightning)

      + P(sensor | ~ burglary ^ lightning) * P(~ burglary) * P(lightning)

      + P(sensor | ~ burglary ^ ~ lightning) * P(~ burglary) * P(~ lightning)
  - > **What do we do now?**

P(sensor) equation – 8

◊ Burglary and lightning are independent

> **We get the following**

» **P(sensor) = P(sensor | burglary ^ lightning)
* P(burglary) * P(lightning)**

**+ P(sensor | burglary ^ ~ lightning)
* P(burglary) * P(~ lightning)**

**+ P(sensor | ~ burglary ^ lightning)
* P(~ burglary) * P(lightning)**

**+ P(sensor | ~ burglary ^ ~ lightning)
* P(~ burglary) * P(~ lightning)**

> **What do we do now?**

– **Substitute the values from the Burglary Probability Table**

P(sensor) evaluation

◊ Substitute the given probabilities

» **P(sensor) = 0.9 * 0.001 * 0.02 (= 0.00001800)**
+ 0.9 * 0.001 * 0.98 (= 0.00088200)
+ 0.1 * 0.999 * 0.02 (= 0.00199800)
+ 0.001 * 0.999 * 0.98 (= 0.00097902)

= 0.00387702

P(sensor) evaluation – 2

◊ Substitute the given probabilities

» **P(sensor) = 0.9 * 0.001 * 0.02 (= 0.00001800)**
+ 0.9 * 0.001 * 0.98 (= 0.00088200)
+ 0.1 * 0.999 * 0.02 (= 0.00199800)
+ 0.001 * 0.999 * 0.98 (= 0.00097902)

= 0.00387702

> **Why did we compute P(sensor)?**
> **What do we do now?**

P(alarm) evaluation – 7b

- The probability equation
  - » P(alarm) = P(alarm I sensor) * P(sensor)
    + P(alarm I ~ sensor) * P(~ sensor)
    - > Evaluate the right hand side
  - » P(alarm I sensor) = 0.95 given
  - » P(alarm I ~ sensor) = 0.001 given
  - » P(~ sensor) = 1 – P(sensor) probability rule
    - > What about P(sensor)?
  - » P(sensor) = 0.00387702 computed

P(alarm) evaluation – 8

- The probability equation
  - P(alarm) = P(alarm I sensor) * P(sensor)
    + P(alarm I ~ sensor) * P(~ sensor)
    - Evaluate the right hand side
  - P(alarm I sensor) = 0.95 given
  - P(alarm I ~ sensor) = 0.001 given
  - P(~ sensor) = 1 – P(sensor) probability rule
    - What about P(sensor)?
  - P(sensor) = 0.00387702 computed
    - What do we do now?

P(alarm) evaluation – 9

- The probability equation
  - » P(alarm) = P(alarm | sensor) * P(sensor)
    + P(alarm | ~ sensor) * P(~ sensor)
    - > Evaluate the right hand side
  - » P(alarm | sensor) = 0.95 given
  - » P(alarm | ~ sensor) = 0.001 given
  - » P(~ sensor) = 1 – P(sensor) probability rule
    - > What about P(sensor)?
  - » P(sensor) = 0.00387702
    - > What do we do now?
      - – Substitute the known values in the probability equation

P(alarm) evaluation – 10

◊ Substitute the given probabilities

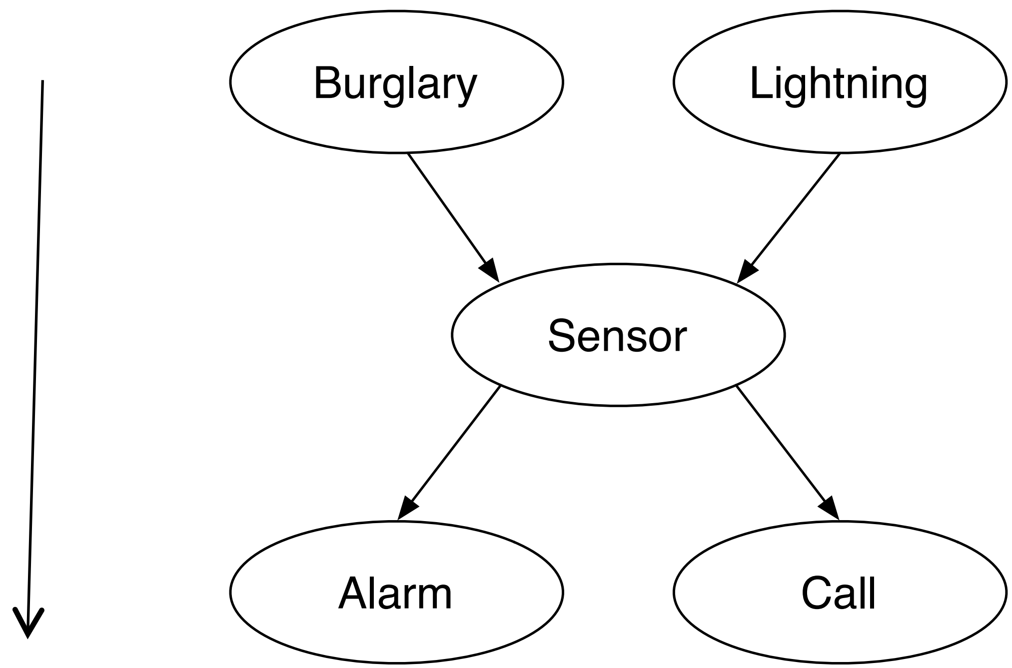
» P(alarm) = P(alarm l sensor) * P(sensor)
+ P(alarm l ~ sensor) * P(~ sensor)

» = 0.95 * 0.00387702 + 0.001 * 0.99612298

» = 0.00467929

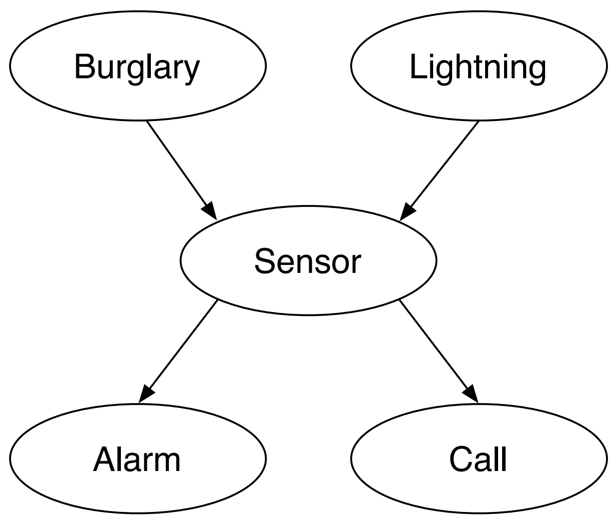
P(burglary l alarm)

◊ The computation uses forward chained reasoning



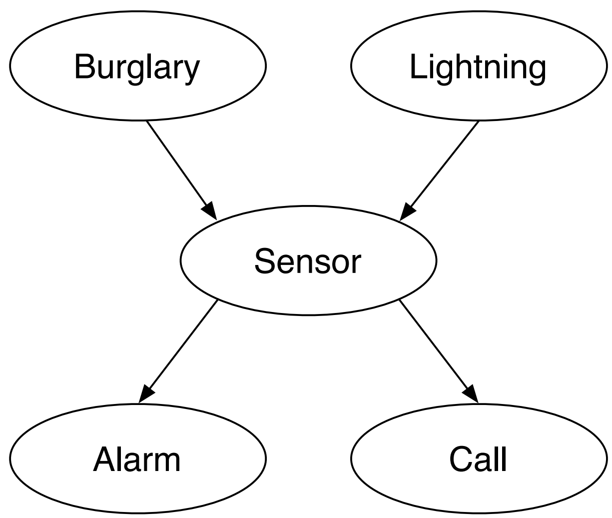
P(burglary I alarm) – 2

- The computation uses forward chained reasoning
  - **The probability computations (arithmetic) can only be done in the forward direction**



P(burglary I alarm) – 3

- The computation uses forward chained reasoning
  - **The probability computations (arithmetic) can only be done in the forward direction**
    - **From known values to unknown values.**



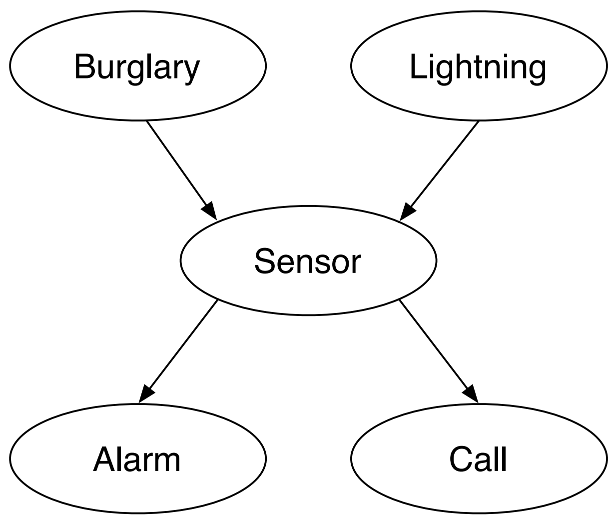
P(burglary I alarm) – 4

◊ The computation uses forward chained reasoning

» **But the probability computations (arithmetic) can only be done in the forward direction**

> **From known values to unknown values.**

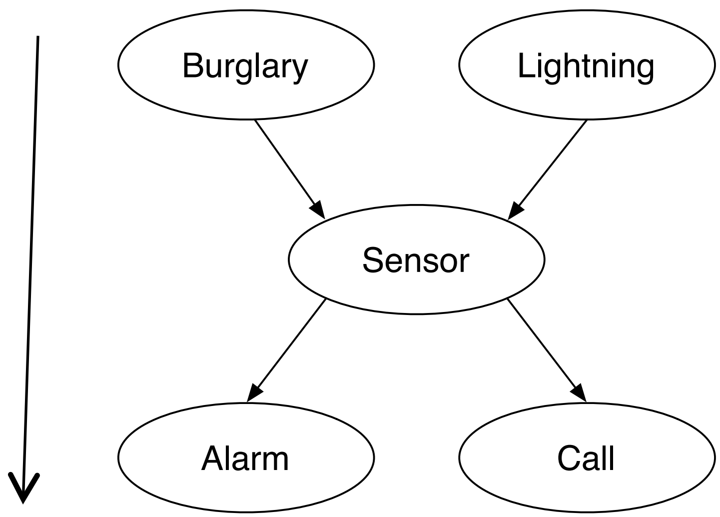
– **So what do we do?**



Bayes' formula

◊ Use Bayes' formula for forward chained reasoning

$$P(Hypothesis \mid Evidence) = P(Hypothesis)\frac{P(Evidence \mid Hypothesis)}{P(Evidence)}$$



Hypothesis = Burglary

Evidence = Alarm

Bayes' formula corollary

- Use Bayes' formula for forward chained reasoning

$$P(Hypothesis \mid Evidence) = P(Hypothesis)\frac{P(Evidence \mid Hypothesis)}{P(Evidence)}$$

- The following is a corollary
  - **H = hypothesis   E = evidence   B = background knowledge**

$$P(H \mid E \wedge B) = P(H \mid B)\frac{P(E \mid H \wedge B)}{P(E \mid B)}$$

P(burglary | alarm) equation

» **What is the probability equation?**

P(burglary | alarm) equation – 3

» **What is the probability equation?**

◊ Use Bayes' formula

» **P(burglary | alarm) = P(burglary) * P(alarm | burglary) / P(alarm)**

$$P(burglary \mid alarm) = P(burglary) * \frac{P(alarm \mid burglary)}{P(alarm)}$$

> **Now what do we do?**

P(burglary I alarm) equation – 4

» **What is the probability equation?**

◊ Use Bayes' formula

» **P(burglary I alarm) = P(burglary) * P(alarm I burglary) / P(alarm)**

$$P(\text{burglary} \mid \text{alarm}) = P(\text{burglary}) * \frac{P(\text{alarm} \mid \text{burglary})}{P(\text{alarm})}$$

> **Now what do we do?**

– **Evaluate the RHS**

P(burglary | alarm) evaluation

- $P(burglary \mid alarm) = P(burglary) * \frac{P(alarm \mid burglary)}{P(alarm)}$
  - What do we know?

P(burglary | alarm) evaluation – 2a

» P(burglary | alarm) = P(burglary) * $\frac{P(\text{alarm} \mid \text{burglary})}{P(\text{alarm})}$

> What do we know?

» P(burglary) = 0.001 given

P(burglary I alarm) evaluation – 2b

» P(burglary I alarm) = P(burglary) * P(alarm I burglary) / P(alarm)

> What do we know?

» P(burglary) = 0.001 given

» P(alarm) = 0.00467929 from an earlier computation

P(burglary | alarm) evaluation – 2c

» P(burglary | alarm) = P(burglary) * $\frac{P(alarm \mid burglary)}{P(alarm)}$

> What do we know?

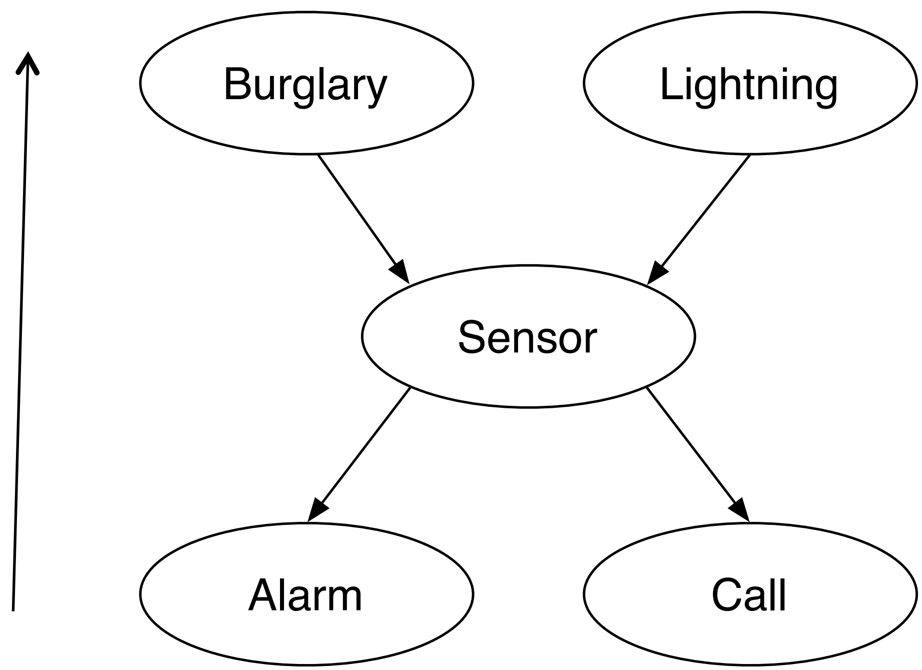
» P(burglary) = 0.001 given

» P(alarm) = 0.00467929 from an earlier computation

» P(alarm | burglary) = ??? need to compute

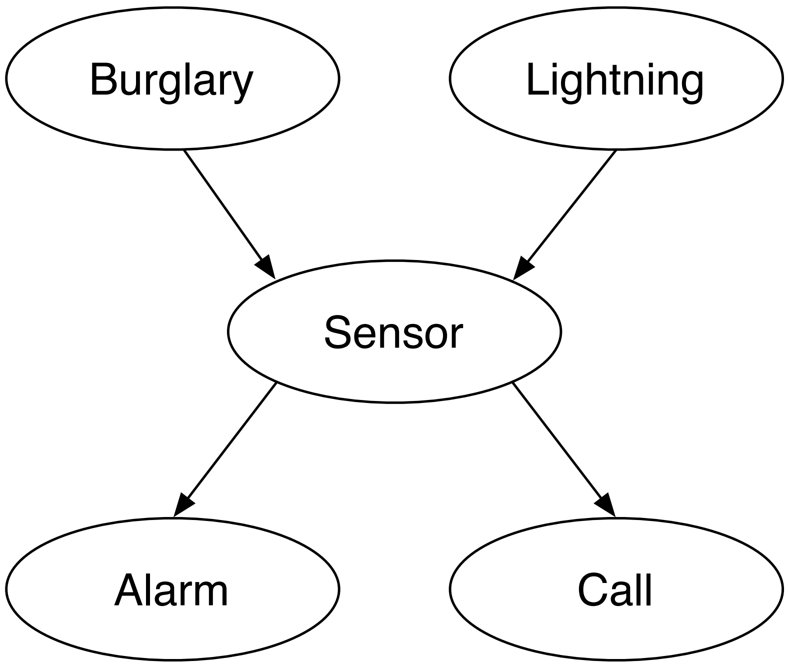
P(alarm | burglary) equation

◊ As before we use backward chained reasoning



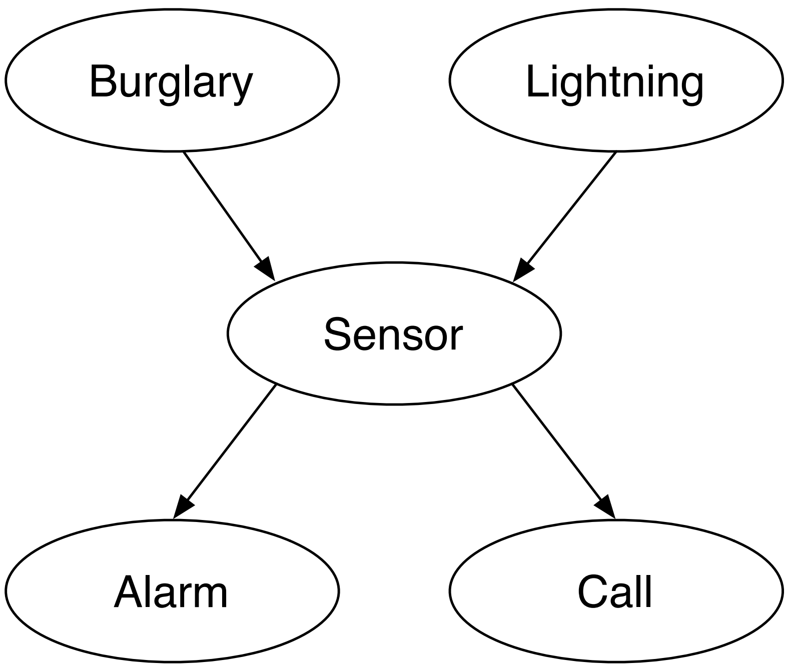
P(alarm | burglary) equation – 2

- ◊ As before we use backward chained reasoning
  - » **What is the probability equation?**



P(alarm | burglary) equation – 3

- As before we use backward chained reasoning
  - » **What is the probability equation?**
  - » **P(alarm | burglary) = P(alarm | sensor) * P(sensor | burglary) + P(alarm | ~ sensor) * P(~ sensor | burglary)**



P(alarm | burglary) evaluation

» P(alarm | burglary) = P(alarm | sensor)
  * P(sensor | burglary)
  +
  P(alarm | ~ sensor)
  * P(~ sensor | burglary)

  > What do we do?

P(alarm | burglary) evaluation – 2

» P(alarm | burglary) = P(alarm | sensor)
 * P(sensor | burglary)
 +
 P(alarm | ~ sensor)
 * P(~ sensor | burglary)

> **What do we do?**

– **Evaluate RHS by substituting known values**

P(alarm | burglary) evaluation – 3

» P(alarm | burglary) = P(alarm | sensor)
  * P(sensor | burglary)
  +
  P(alarm | ~ sensor)
  * P(~ sensor | burglary)

» P(alarm | sensor) = 0.95 given

P(alarm l burglary) evaluation – 4

» P(alarm l burglary) = P(alarm l sensor)
  * P(sensor l burglary)
  +
  P(alarm l ~ sensor)
  * P(~ sensor l burglary)

» P(alarm l sensor) = 0.95 given

» P(alarm l ~ sensor) = 0.001 given

P(alarm | burglary) evaluation – 5

» P(alarm | burglary) = P(alarm | sensor)
  * P(sensor | burglary)
  +
  P(alarm | ~ sensor)
  * P(~ sensor | burglary)

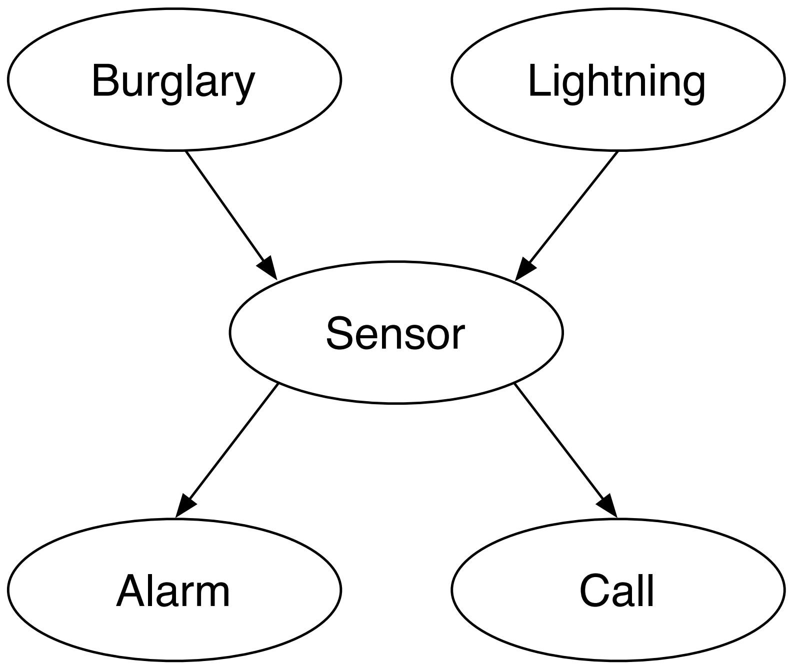
» P(alarm | sensor) = 0.95 given

» P(alarm | ~ sensor) = 0.001 given

» P(sensor | burglary) = ??? need to compute

P(sensor | burglary) equation

» **What is the probability equation?**



P(sensor | burglary) equation – 2

◊ From an earlier slide we have

» P(sensor) = P(sensor | burglary ^ lightning) * P(burglary) * P(lightning)

+ P(sensor | burglary ^ ~ lightning) * P(burglary) * P(~ lightning)

+ P(sensor | ~ burglary ^ lightning) * P(~ burglary) * P(lightning)

+ P(sensor | ~ burglary ^ ~ lightning) * P(~ burglary) * P(~ lightning)

> What do we do now?

P(sensor | burglary) equation – 3

- From an earlier slide we have
  - » **P(sensor) = P(sensor | burglary ^ lightning)**
    *** P(burglary) * P(lightning)**

    + P(sensor | burglary ^ ~ lightning)
    *** P(burglary) * P(~ lightning)**

    + P(sensor | ~ burglary ^ lightning)
    *** P(~ burglary) * P(lightning)**

    + P(sensor | ~ burglary ^ ~ lightning)
    *** P(~ burglary) * P(~ lightning)**
    - > **What do we do now?**
      - – **Simplify the equation based on the evidence**

P(sensor | burglary) equation – 4

- ◊ Substitute P(burglary) = 1 and P(~ burglary) = 0
  - » **P(sensor) = P(sensor | burglary ^ lightning) * P(lightning)**

    + P(sensor | burglary ^ ~ lightning) * P(~ lightning)

    - > **What do we do now?**

P(sensor | burglary) equation – 5

- ◊ Substitute P(burglary) = 1 and P(~ burglary) = 0
  - » **P(sensor) = P(sensor | burglary ^ lightning)**
    *** P(lightning)**
    + P(sensor | burglary ^ ~ lightning)
    *** P(~ lightning)**
- ◊ Substitute the known values
  - » **P(sensor) = 0.9 * 0.02 + 0.9 * 0.98**
    = 0.9

P(sensor | burglary) equation – 6

◊ Substitute P(burglary) = 1 and P(~ burglary) = 0

» P(sensor) = P(sensor | burglary ^ lightning)
  * P(lightning)

  + P(sensor | burglary ^ ~ lightning)
  * P(~ lightning)

◊ Substitute the known values

» P(sensor) = 0.9 * 0.02 + 0.9 * 0.98
  = 0.9

> What do we do now?

P(alarm | burglary) evaluation – 7

» P(alarm | burglary) = P(alarm | sensor)
 * P(sensor | burglary)
 +
 P(alarm | ~ sensor)
 * P(~ sensor | burglary)

» P(alarm | sensor) = 0.95 given

» P(alarm | ~ sensor) = 0.001 given

» P(sensor | burglary) = 0.9 from computation

P(alarm | burglary) evaluation – 7

» P(alarm | burglary) = P(alarm | sensor)
  * P(sensor | burglary)
  +
  P(alarm | ~ sensor)
  * P(~ sensor | burglary)

» P(alarm | sensor) = 0.95 given

» P(alarm | ~ sensor) = 0.001 given

» P(sensor | burglary) = 0.9 from computation

> What is missing?
> How do we compute it?

P(alarm | burglary) evaluation – 8

» P(alarm | burglary) = P(alarm | sensor)
  * P(sensor | burglary)
  +
  P(alarm | ~ sensor)
  * P(~ sensor | burglary)

» P(alarm | sensor) = 0.95 given

» P(alarm | ~ sensor) = 0.001 given

» P(sensor | burglary) = 0.9 from computation

» P(~ sensor | burglary) = 1 – P(sensor | burglary)
  = 0.1

P(alarm | burglary) evaluation – 9

- » P(alarm | burglary) = P(alarm | sensor)
  * P(sensor | burglary)
  \+
  P(alarm | ~ sensor)
  * P(~ sensor | burglary)

- » P(alarm | sensor) = 0.95 given
- » P(alarm | ~ sensor) = 0.001 given
- » P(sensor | burglary) = 0.9 from computation
- » P(~ sensor | burglary) = 1 – P(sensor | burglary)
  = 0.1
  - > What do we do now?

P(alarm | burglary) evaluation – 10

- » P(alarm | burglary) = P(alarm | sensor)
  * P(sensor | burglary)
  \+
  P(alarm | ~ sensor)
  * P(~ sensor | burglary)

- » P(alarm | sensor) = 0.95 given
- » P(alarm | ~ sensor) = 0.001 given
- » P(sensor | burglary) = 0.9 from computation
- » P(~ sensor | burglary) = 1 – P(sensor | burglary)
  = 0.1

◊ Substitute the known values

- » P(alarm | burglary) = 0.8551

P(burglary l alarm) evaluation – 3

- We have the following from “evaluation – 2”
  - » P(burglary l alarm) = P(burglary) * $\frac{\text{P(alarm l burglary)}}{\text{P(alarm)}}$
  - » P(burglary) = 0.001 given
  - » P(alarm) = 0.00467929 from an earlier computation
  - » P(alarm l burglary) = ??? need to compute
- Now we know
  - » P(alarm l burglary) = 0.8551
  - » P(burglary l alarm) = 0.001 * (0.8551 / 0.00467929)
    = 0.1827414

Probability reasoning rules

◊ (1) Probability of a conjunction

$$P(N1 \wedge N2 \mid C) = P(N1 \mid C) \times P(N2 \mid N1 \wedge C)$$

◊ (2) Probability of a certain event

$$P(N \mid N \wedge C1 \wedge \ldots) = 1$$

◊ (3) Probability of an impossible event

$$P(N \mid \sim N \wedge C1 \wedge \ldots) = 0$$

◊ (4) Probability of a negation

$$P(\sim N \mid C) = 1 - P(N \mid C)$$

Probability reasoning rules – 2

- ◊ (5) If the condition involves a descendant of N use the corollary (general form) of Bayes' theorem

$$P(N \mid D \wedge C) = P(N \mid C) \times P(D \mid N \wedge C) / P(D \mid C)$$

- ◊ (6) If the condition does not involve a descendent of N then

- ◊ (6a) If N has no parents

$$P(N \mid C) = P(N)$$ **Given in the model**

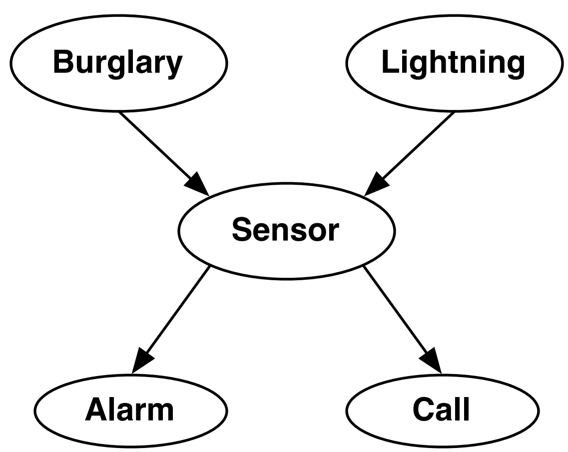
- ◊ (6b) If N has parents

$$P(N \mid C) = \sum_{S \in states(parents)} P(N \mid S) \times P(S \mid C)$$

P(call | alarm) equation

- The probability equation
  - » $P(call \mid alarm) = P(call \mid sensor) * P(sensor \mid alarm)$
    $+ P(call \mid \sim sensor) * P(\sim sensor \mid alarm)$
    $= 0.70841$

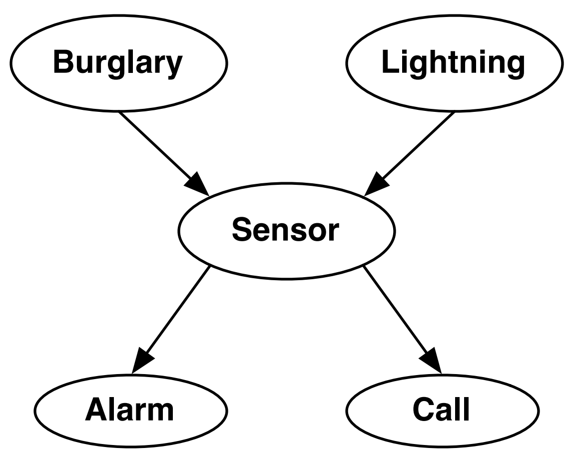
    $P(call \mid sensor) = 0.95$ given
    $P(call \mid \sim sensor) = 0.0$ given
    $P(sensor \mid alarm) = 0.78712$ see next slide
    $P(\sim sensor \mid alarm) = 1 - P(sensor \mid alarm) = 0.21288$



P(sensor | alarm) equation

- The probability equation
  - » **P(sensor | alarm) = P(sensor) * P(alarm | sensor) / P(alarm)**

P(sensor) = 0.00387702 **see previous slides**
P(alarm | sensor) = 0.95 **given**
P(alarm) = 0.467929 **see previous slides**

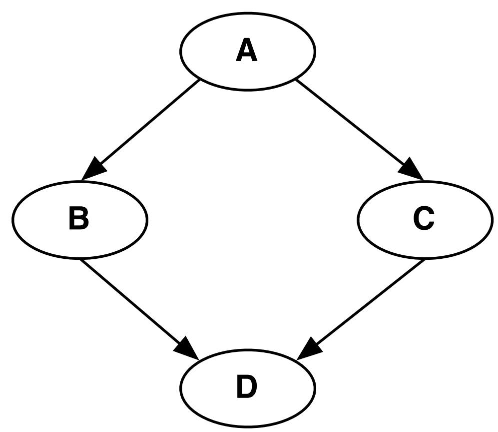


P(D) equation

- The probability equation
  - P(D) = P(D | B, C) * P(B, C) + P(D | B, ~C) * P(B, ~C) + P(D | ~B, C) * P(~B, C) + P(D | ~B, ~C) * P(~B, ~C)   **Given in the model**

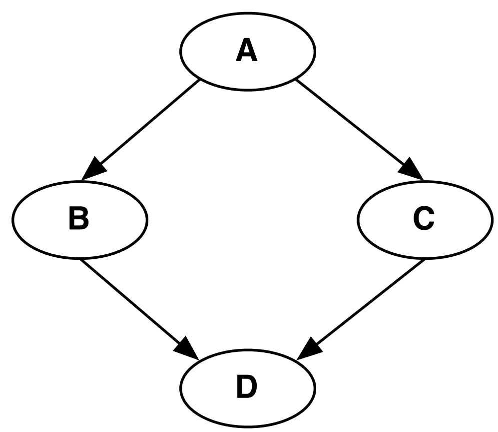
    > **Here B and C are dependent**

    – **Cannot simplify as we did with the P(sensor)**



P(D) equation – 2

- ◊ The probability equations, level 2
  - » $P(B, C) = P(B) * P(C \mid B)$
  - » $P(B, \sim C) = P(B) * P(\sim C \mid B)$
    $= P(B) * 1 - P(C \mid B)$
  - » $P(\sim B, C) = P(\sim B) * P(C \mid \sim B)$
    $= (1 - P(B)) * P(C \mid \sim B)$
  - » $P(\sim B, \sim C) = P(\sim B) * P(\sim C \mid \sim B)$
    $= (1 - P(B)) * (1 - P(C \mid \sim B))$
  - » $P(B) = P(B \mid A) * P(A) + P(B \mid \sim A) * P(\sim A)$
    Given in the model



P(D) equation – 3

- The probability equations, level 3
  - » $P(C | B) = P(C | A) * P(A | B) + P(C | \sim A) * P(\sim A | B) = P(C | \sim A) * (1 - P(A | B))$
  - » $P(C | \sim B) = P(C | A) * P(A | \sim B) + P(C | \sim A) * P(\sim A | \sim B) = P(C | \sim A) * (1 - P(A | \sim B))$
  - » $P(A | B) = P(A) * P(B | A) / P(B)$
  - » $P(A | \sim B) = P(A) * P(\sim B | A) / P(\sim B) = P(A) * P(\sim B | A) / (1 - P(B))$

Given in the model or computed previously