# Bayesian Networks
# Part 1 of 4
# Why have them
# What are they

# Alternate names

- Bayesian networks are also called by the following names
  - **Belief network**
  - **Probabilistic network**
  - **Causal network**

# Why

◊ Need causal and explanatory models for risk assessment

# Why – Medical

◊ Need causal and explanatory models for risk assessment

» **As a doctor or a patient, how do you arrive at a decision of what to do on the basis of symptoms, diagnostic tests and effectiveness of different treatments?**

# Why – Medical – Example

- One in a thousand people has a prevalence for a particular heart disease

# Why – Medical – Example – 2

- One in a thousand people has a prevalence for a particular heart disease
- There is a test to detect this disease. The test is 100% accurate for people who have the disease

# Why – Medical – Example – 3

- One in a thousand people has a prevalence for a particular heart disease
- There is a test to detect this disease.  The test is 100% accurate for people who have the disease
- The test is 95% accurate for those who don't have the disease

# Why – Medical – Example – 4

- One in a thousand people has a prevalence for a particular heart disease
- There is a test to detect this disease.  The test is 100% accurate for people who have the disease
- The test is 95% accurate for those who don't have the disease
  - **This means that 5% of people who do not have the disease will be wrongly diagnosed as having it**

# Why – Medical – Example – 5

- One in a thousand people has a prevalence for a particular heart disease
- There is a test to detect this disease. The test is 100% accurate for people who have the disease
- The test is 95% accurate for those who don't have the disease
  - » **This means that 5% of people who do not have the disease will be wrongly diagnosed as having it**
- If a randomly selected person tests positive what is the probability that the person actually has the disease?

# Why – Medical – Example – 6

◊ If a randomly selected person tests positive what is the probability that the person actually has the disease?

» **Students and staff at the Harvard Medical School were asked this question**

# Why – Medical – Example – 7

- If a randomly selected person tests positive what is the probability that the person actually has the disease?
  - **Students and staff at the Harvard Medical School were asked this question**
    - **Half gave the response 95%**
    - **The ‘average’ answer was 56%**

# Why – Medical – Example – 8

◊ If a randomly selected person tests positive what is the probability that the person actually has the disease?

» **Students and staff at the Harvard Medical School were asked this question**

> **Half gave the response 95%**

> **The 'average' answer was 56%**

» **The real answer is just under 2%**

# Why – Medical – Example – 9

◊ If a randomly selected person tests positive what is the probability that the person actually has the disease?

» **Students and staff at the Harvard Medical School were asked this question**

> **Half gave the response 95%**

> **The 'average' answer was 56%**

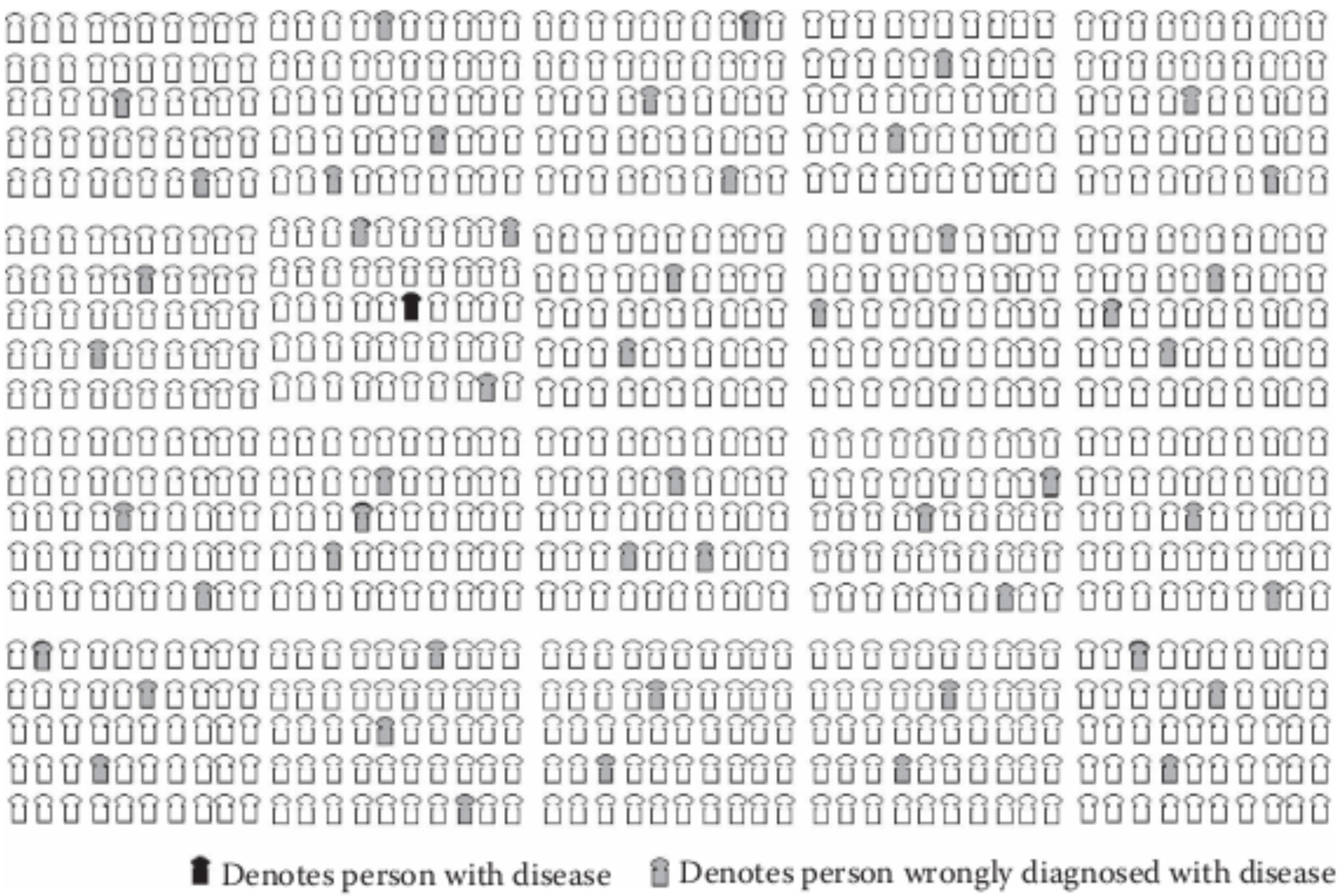
» **The real answer is just under 2%**

» **What went wrong?**

# Why – Medical – Example – 10

- If a randomly selected person tests positive what is the probability that the person actually has the disease?
  - **Students and staff at the Harvard Medical School were asked this question**
    - **Half gave the response 95%**
    - **The 'average' answer was 56%**
  - **The real answer is just under 2%**
  - **What went wrong?**
    - **Ignore the base rate of the disease**

# Why – Medical – Example diagram




*The Signal and the Noise*, Nate Silver, Penguin Press, 2012, p246

# Why – Legal

- Need causal and explanatory models for risk assessment

  » **As a member of a jury how do you weigh the evidence for and against the guilt of the defendant?**

# Why – Legal – Example

◊ The chances that an innocent person has the matching blood type is 1/1000.

# Why – Legal – Example – 2

- The chances that an innocent person has the matching blood type is 1/1000.
- Fred, a random person from a town with 10,000 people, has the matching blood type.

# Why – Legal – Example – 3

- The chances that an innocent person has the matching blood type is 1/1000.
- Fred, a random person from a town with 10,000 people, has the matching blood type.
- Therefore the chances that Fred is
  - **Innocent is just 1 / 1000 (0.1%)**
    - **Or**
  - **Guilty with probability of 999 / 1000 (99.9%).**

# Why – Legal – Example – 4

- ◊ The chances that an innocent person has the matching blood type is 1/1000.
- ◊ Fred, a random person from a town with 10,000 people, has the matching blood type.
- ◊ Therefore the chances that Fred is
  - » **Innocent is just 1 / 1000 (0.1%)**
    - > **Or**
  - » **Guilty with probability of 999 / 1000 (99.9%).**
    - > **The real answer**
  - » **Innocent with ≃ 10 / 11 probability**

# Why – Legal – Example – 5

◊ What went wrong?

# Why – Legal – Example – 6

◊ What went wrong?

» **In a town with 10,000 people there are about 10 other people with matching blood type.**

# Why – Legal – Example – 7

- What went wrong?
  - » **In a town with 10,000 people there are about 10 other people with matching blood type.**
    - > **1 guilty person**
    - > **And 10 out of 9,999 innocent people with a matching test**

# Why – Legal – Example – 8

- What went wrong?
  - In a town with 10,000 people there are about 10 other people with matching blood type.
    - 1 guilty person
    - And 10 out of 9,999 innocent people with a matching test
  - So there is only a 9% chance, 1 / 11, that Fred is guilty
    - And a 91% chance that he is innocent.

# Why – Legal – Example – 9

- This example is known as the
  - **Prosecutor's Fallacy**

# Why – Legal – Example – 10

- This example is known as the
  - **Prosecutor's Fallacy**
- There is a corresponding
  - **Defendant's Fallacy**

# Why – Legal – Defendant's Fallacy

- The evidence presented by the prosecution leads us to conclude that there is actually a very high probability that the defendant is innocent.  Therefore this evidence is worthless even for the prosecutor's argument and so can safely be ignored.

# Why – Legal – Defendant's Fallacy – 2

- ◊ The evidence presented by the prosecution leads us to conclude that there is actually a very high probability that the defendant is innocent. Therefore this evidence is worthless even for the prosecutor's argument and so can safely be ignored.

  » **The argument is wrong because the evidence has moved our belief in Fred being at the scene from 1/1,000 to about 9/100. A significant change that cannot be ignored.**

# Why – Safety

◊ Need causal and explanatory models for risk assessment

» **How do we determine the risk of flood by taking into account existing defensive measures, amount of rainfall and current river level?**

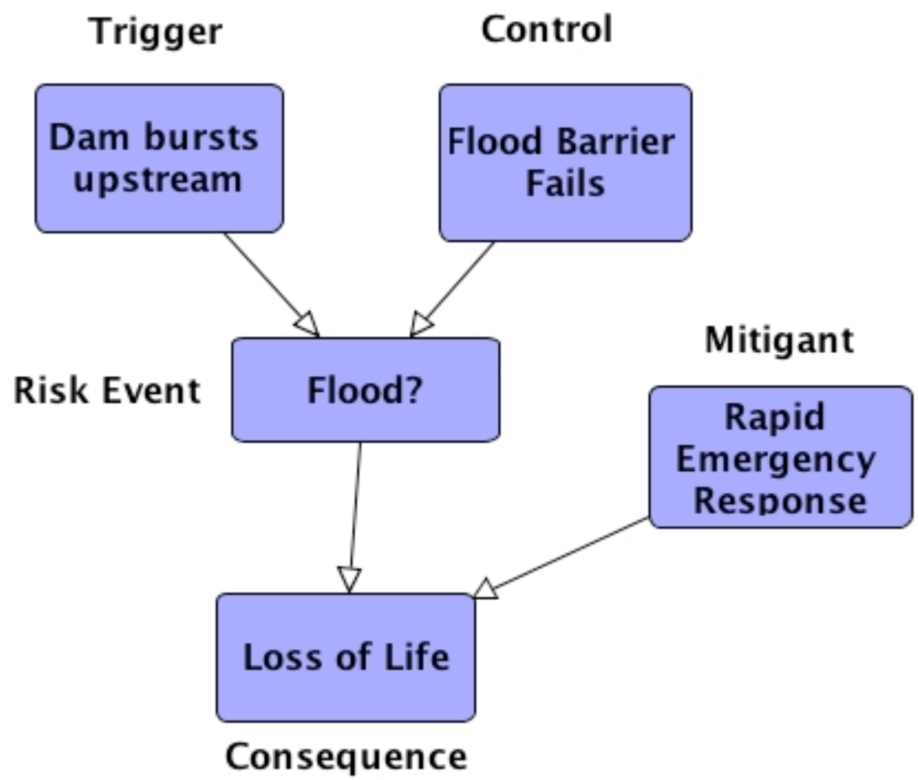
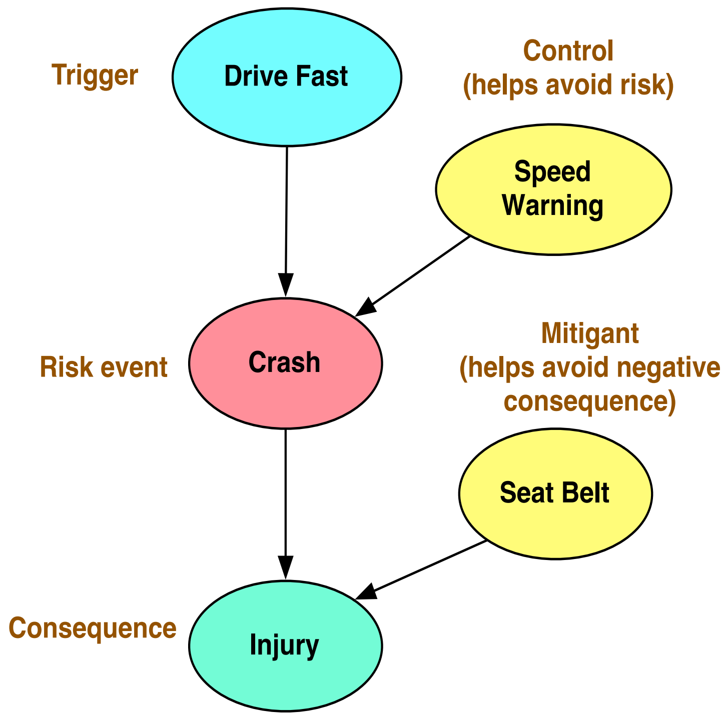



*Figure from*
*Risk Assessment and Decision Analysis with Bayesian Networks*, Norman Fenton, Martin Neil, CRC Press, 2013, p43

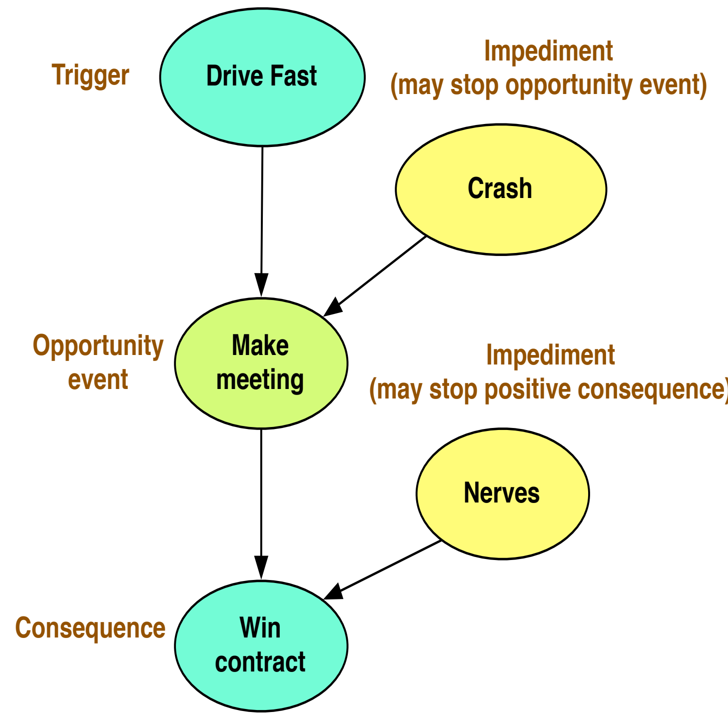
# Why – Reliability

- The **success** or **failure** of new products and systems that depend upon their reliability, as experienced by end-users

# Why – Risk & Opportunity



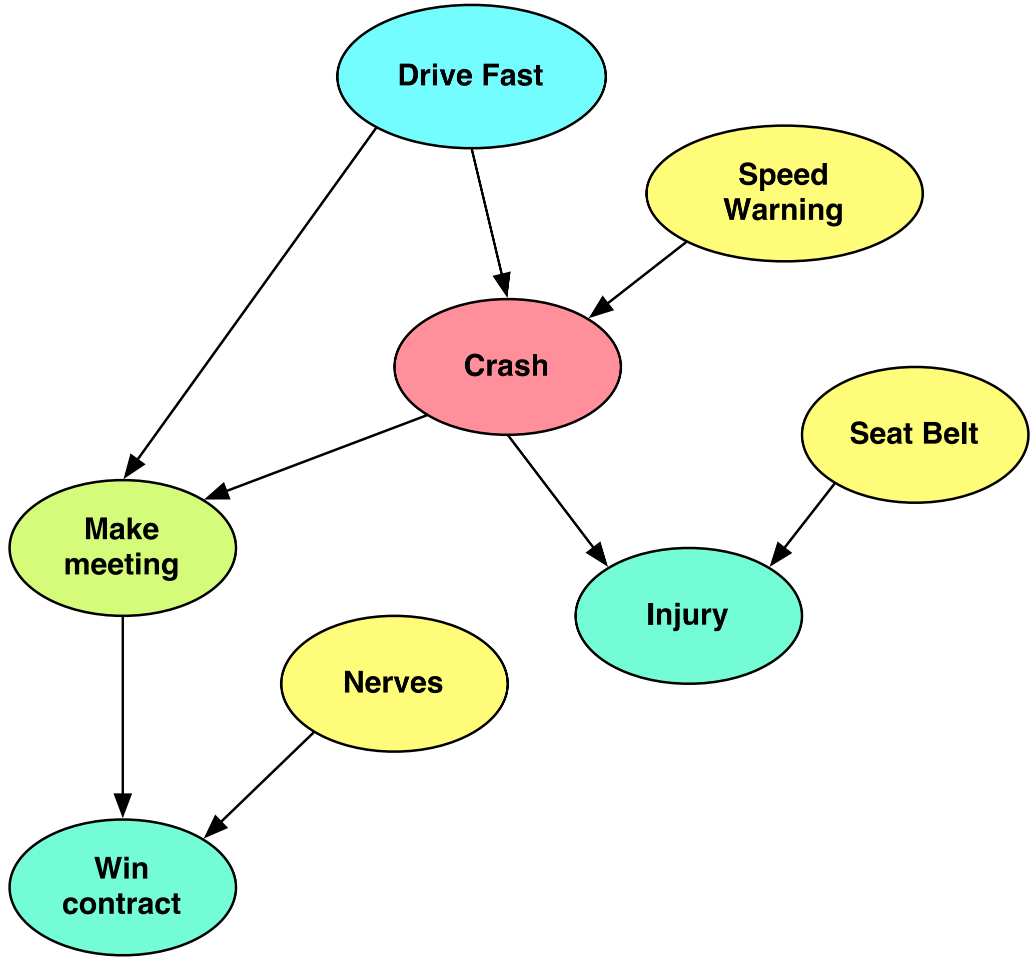

Use BN for risk analysis




Use BN for opportunity analysis

*Risk Assessment and Decision Analysis with Bayesian Networks*,
Norman Fenton, Martin Neil, CRC Press, 2013, p42

# Why – Risk & Opportunity – 2




Best is to combine both risk and opportunity analysis

*Risk Assessment and Decision Analysis with Bayesian Networks*,
Norman Fenton, Martin Neil, CRC Press, 2013, p43

# Other Bayesian processes

◊ Playing chess

» **Following the most promising line of attack**

# Other Bayesian processes – 2

◊ Playing chess

» **Following the most promising line of attack**

◊ Betting on sports

» **Follow latest trade and injury news**

# Other Bayesian processes – 3

◊ Playing chess

» **Following the most promising line of attack**

◊ Betting on sports

» **Follow latest trade and injury news**

◊ Forecasting

» **Weather, Economy, Stock Market**

» **War, Peace, Terrorist attacks**

# Other Bayesian processes – 4

**In a partial information system, how much are you willing to bet on the conclusions reached as a result of your analysis**

# Other Bayesian processes – 5

**In a partial information system, how much are you willing to bet on the conclusions reached as a result of your analysis**

**As you gather more information you update your analysis, increasing the reliability of your conclusions**

# Terrorist example

◊ Before the first plane hit the World Trade Center the probability of it being caused by terrorists would be very low.

*Example from*
*The Signal and the Noise*, Nate Silver, Penguin Press, 2012, pp247..248

# Terrorist example – 2

- Before the first plane hit the World Trade Center the probability of it being caused by terrorists would be very low.
  - **Suppose it to be 0.005%**

# Terrorist example – 3

◊ Before the first plane hit the World Trade Center the probability of it being caused by terrorists would be very low.

» **Suppose it to be 0.005%**

◊ We run our model

» **The probability rises to 38%**

# Terrorist example – 4

◊ Before the first plane hit the World Trade Center the probability of it being caused by terrorists would be very low.

  » **Suppose it to be 0.005%**

◊ We run our model

  » **The probability rises to 38%**

◊ We put 38% into our model, and run it again

  » **The probability rises to 99.987%**

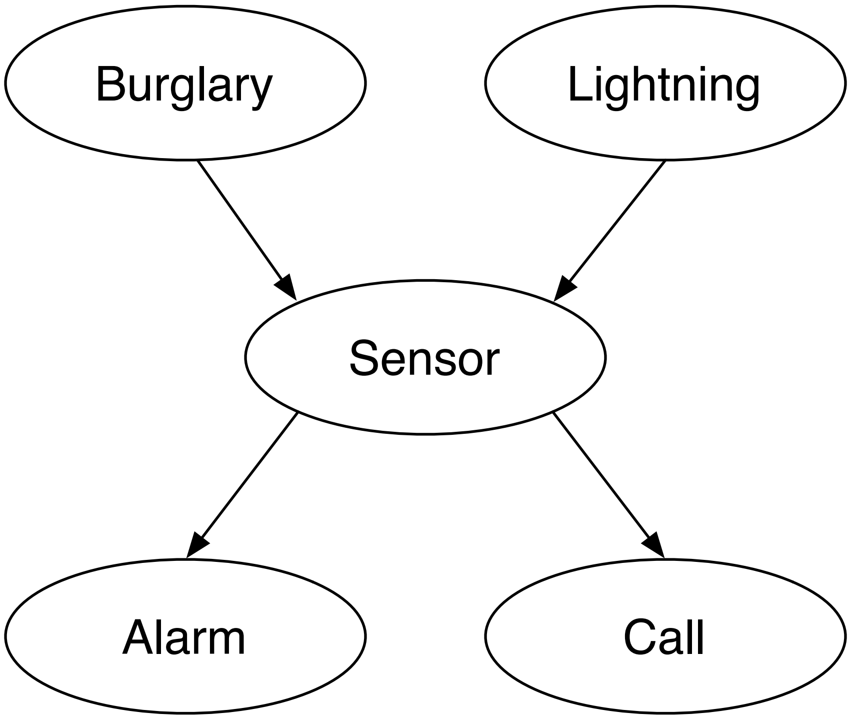
# Terrorist example – 5

- ◊ Before the first plane hit the World Trade Center the probability of it being caused by terrorists would be very low.
  - » **Suppose it to be 0.005%**
- ◊ We run our model
  - » **The probability rises to 38%**
- ◊ We put 38% into our model, and run it again
  - » **The probability rises to 99.987%**
    - > **A virtual certainty**

# Burglary Bayesian model

The occurrence of events burglary or/and lightning can cause the event sensor to occur

What kind of graph is this?

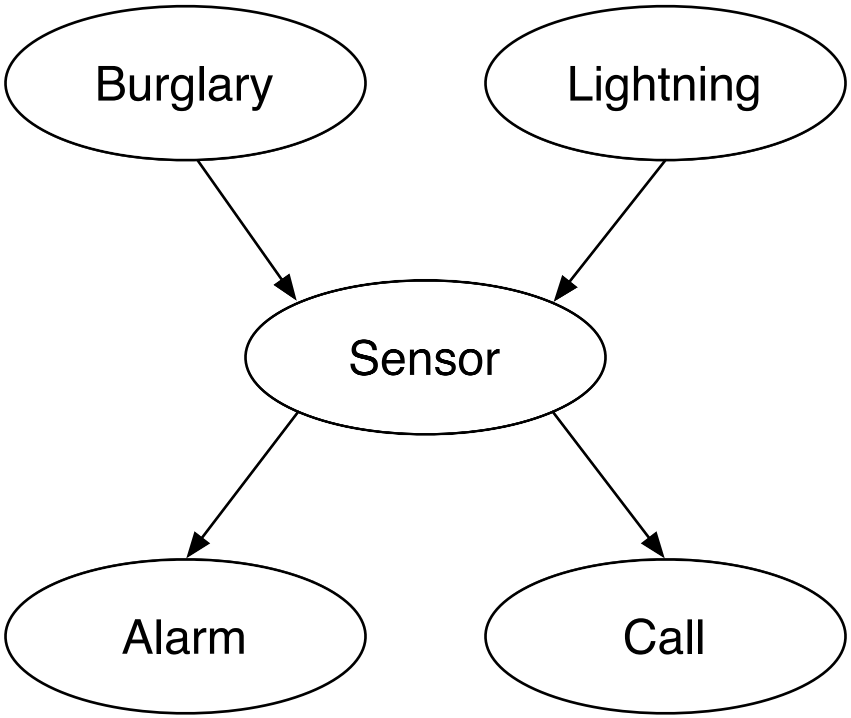



When event sensor occurs, the alarm and call events may occur

# Burglary Bayesian model

The occurrence of events burglary or/and lightning can cause the event sensor to occur

Model Is a DAG (directed acyclic graph)

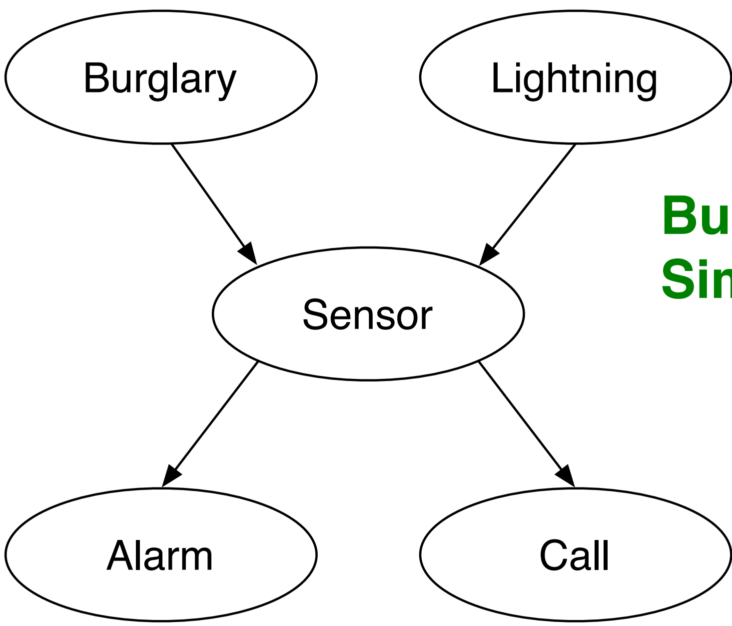



When event sensor occurs, the alarm and call events may occur

# Burglary Bayesian model – 2

**Every node (event, variable) has a probability table associated with it that gives the probability of the event occurring**

| Burglary variable | | Lightning variable | |
|---|---|---|---|
| True | 0.1% | True | 2% |
| False | 99.9% | False | 98% |




**Burglary and lightning have no parents. Simple table**

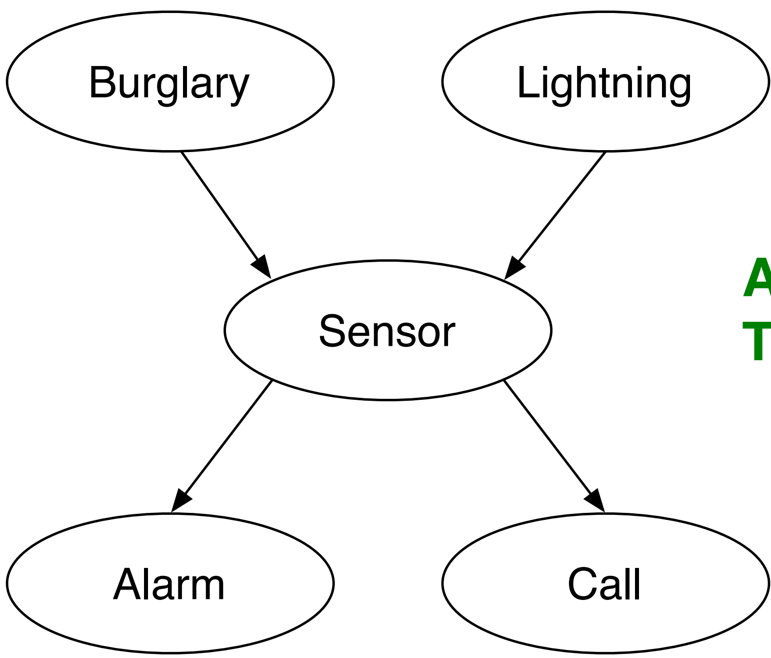
# Burglary Bayesian model – 3

**Alarm variable**

| Sensor | True | False |
|---|---|---|
| True | 95% | 0.1% |
| False | 5% | 99.9% |

**Call variable**

| Sensor | True | False |
|---|---|---|
| True | 90% | 0% |
| False | 10% | 100% |

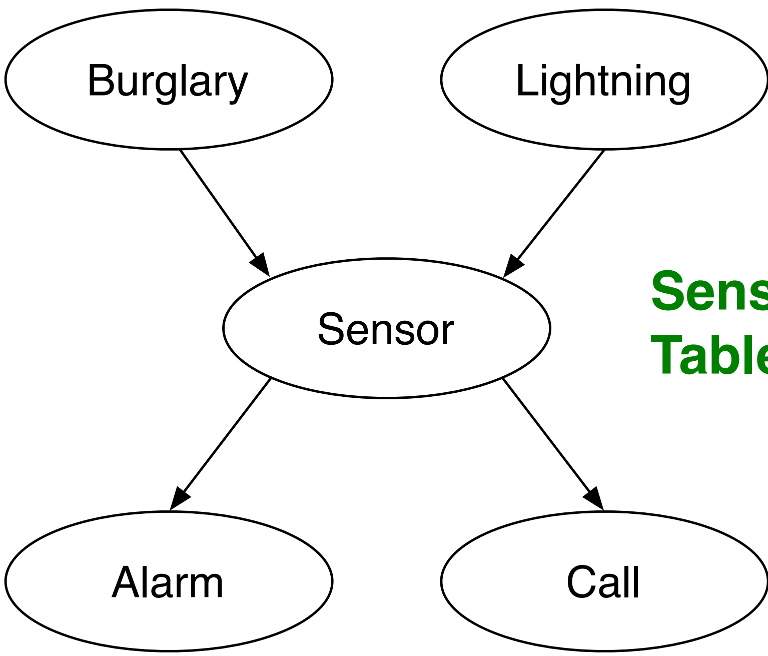



**Alarm and call have one parent each.**
**Table depends upon state of parent**

# Burglary Bayesian model – 4

**Sensor variable**

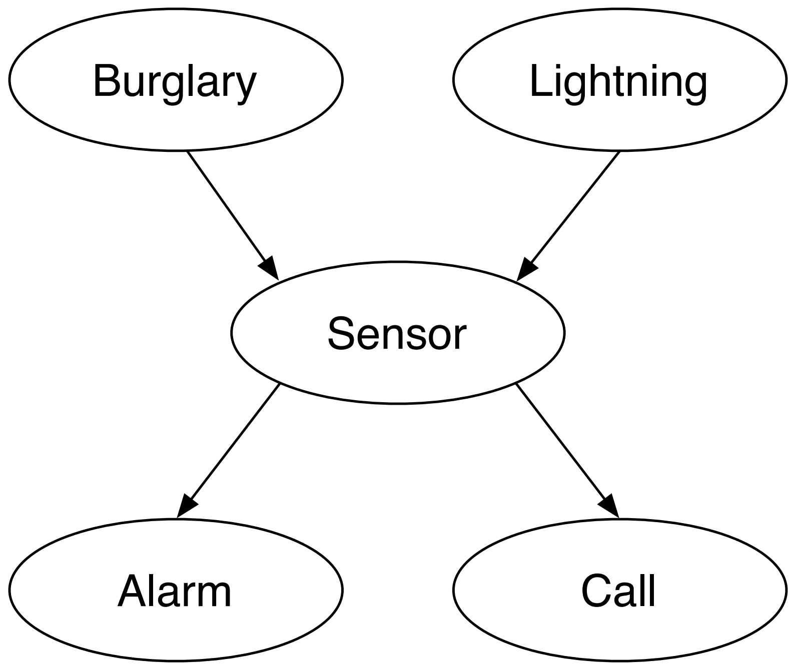
| Burglary | True | | False | |
|---|---|---|---|---|
| Lightning | True | False | True | False |
| True | 90% | 90% | 10% | 0.1% |
| False | 10% | 10% | 90% | 99.9% |




**Sensor has two parents.**
**Table depends upon state of every parent**

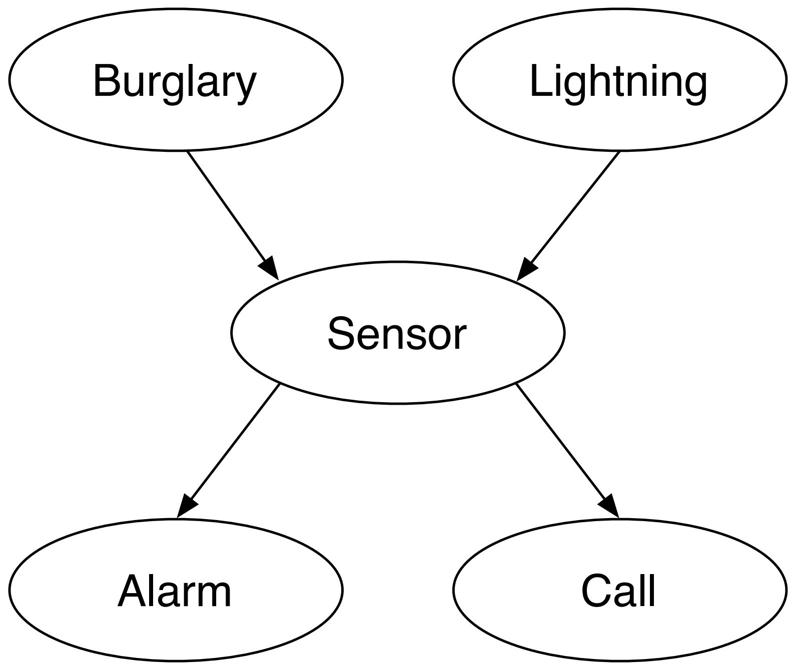
# Burglary model question 1

- If the alarm goes off, what is the probability of it being caused by
  - **Burglary?** **Lightning?** **Both?** **Neither?**



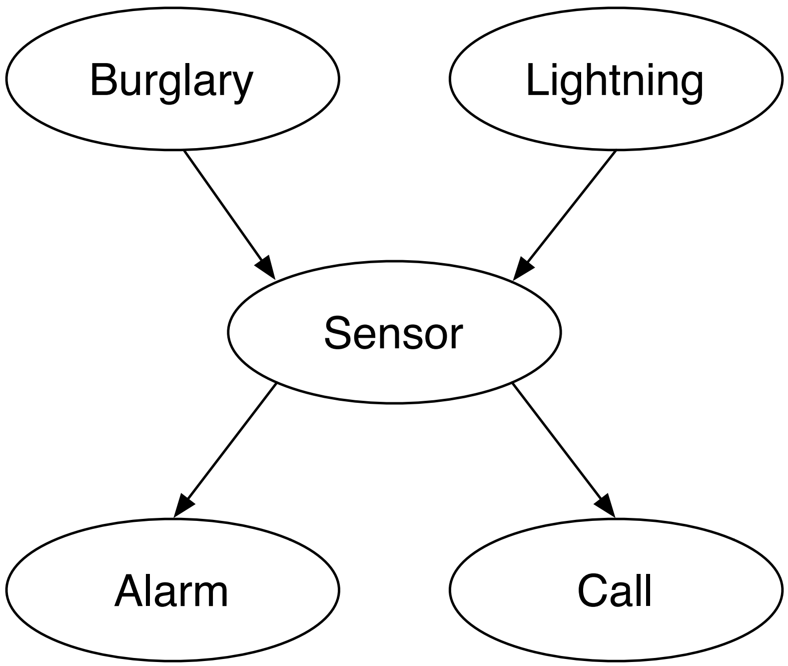
# Burglary model question 2

◊ If a burglary occurs, what is the probability of the alarm sounding?



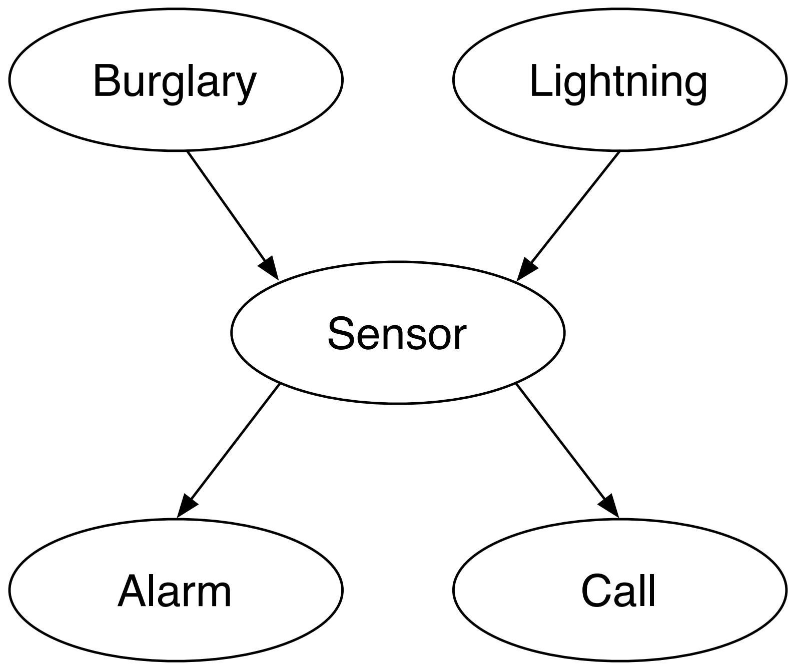
# Burglary model question 3

◊ What is the probability of not getting a call if there is a burglary?



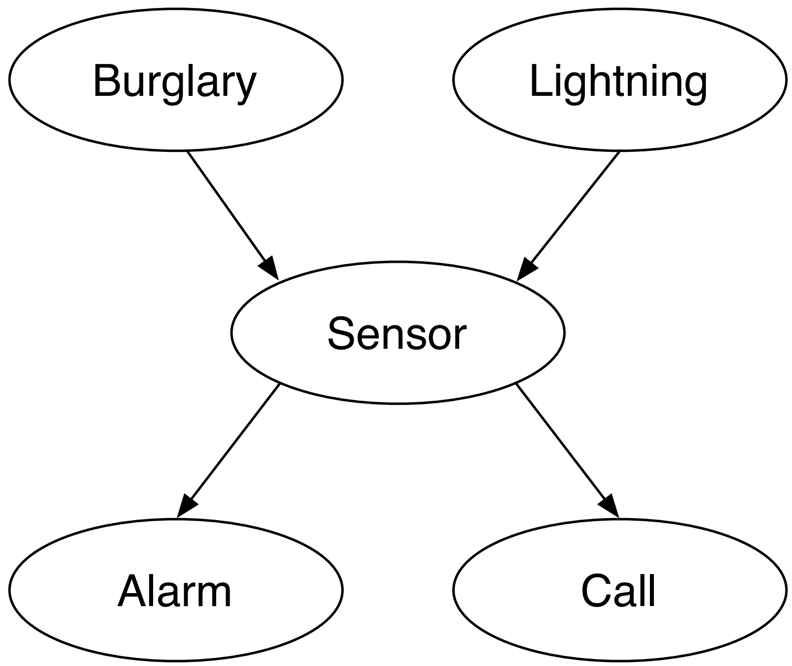
# Burglary model question – 4

◊ What is the probability of getting a call, if there is lightning?



# Burglary model question – 5

◊ What is the probability of getting both a call and an alarm, if there is a burglary?



# Definition of a proposition

- **What is a proposition?**

# Definition of a proposition – 2

- A proposition X is a statement that can be either true or false.

# Example proposition 1a

◊ Alice is a character in “Alice in Wonderland”

# Example proposition 1b

- ◊ Alice is a character in “Alice in Wonderland”
  - » **Happens to be true**

# Example proposition 2a

◊ Tom is a character in “Alice in Wonderland”

# Example proposition 2b

- Tom is a character in “Alice in Wonderland”
  - **Happens to be false**

# Definition of a proposition – 3

◊ If X and Y are propositions

» **Then what else are propositions?**

# Definition of a proposition – 4

◊ If X and Y are propositions, then the following are also propositions

» $X \wedge Y$ the conjunction of X and Y

» $X \vee Y$ the disjunction of X and Y

» $\sim X$ the negation of X

# Proposition probability

◊ Propositions can not only be true or false but can have a probability of being true

# Proposition probability– 2

- Propositions can not only be true or false but can have a probability of being true
  - **A level of belief in the truth of the statement.**

# Proposition probability– 3

◊ Propositions can not only be true or false but can have a probability of being true

» **A level of belief in the truth of the statement**

> **How much are you willing to bet on the truth of the proposition**

# Probability notation

◊ Propositions can not only be true or false but can have a probability of being true; a level of belief in the truth of the statement.

   » **P(X)** **the probability that X is true**

# Example proposition 3a

◊ Alice has a cold while she is in Wonderland.

# Example proposition 3b

- ◊ Alice has a cold while she is in Wonderland.
  - » **Don't know, could be true or false**
    - > **Assign a probability**
    - > **A level of belief**

# Example proposition 4a

- ◊ It will rain on 2025 June 16

# Example proposition 4b

- ◊ It will rain on 2025 June 16
  - » **Don’t know, could be true or false**
    - > **Assign a probability**
    - > **A level of belief**

# Probability notation – 2a

◊ Propositions can not only be true or false but can have a probability of being true; a level of belief in the truth of the statement.

» **P(X)** **the probability that X is true**

» **P(X | Y)** **the probability that X is true, assuming Y is true**

# Probability notation – 2b

◊ Propositions can not only be true or false but can have a probability of being true; a level of belief in the truth of the statement.

» **P(X)** **the probability that X is true**

» **P(X | Y)** **the probability that X is true, assuming Y is true**

> **Y is thought of as evidence**

# Probability notation – 2c

◊ Propositions can not only be true or false but can have a probability of being true; a level of belief in the truth of the statement.

- **P(X)** **the probability that X is true**
- **P(X | Y)** **the probability that X is true, assuming Y is true**
  - **Y is thought of as evidence**
  - **Or background knowledge**

# Probability notation – 2d

◊ Propositions can not only be true or false but can have a probability of being true; a level of belief in the truth of the statement.

- » **P(X)** **the probability that X is true**
- » **P(X I Y)** **the probability that X is true, assuming Y is true**
  - > **Y is thought of as evidence**
  - > **Or background knowledge**
  - > **Or prior belief**

# Probability with evidence example 1.1

- Let X be “the moon is made of green cheese”
  Let Y be “there are mice on the moon”

# Probability with evidence example 1.2

◊ Let X be “the moon is made of green cheese”
Let Y be “there are mice on the moon”

» **P(X I Y) – probability “the moon is made of green cheese” is true given the evidence “there are mice on the moon” is true**

# Probability with evidence example 1.3

◊ Let X be "the moon is made of green cheese"
Let Y be "there are mice on the moon"

- » **P(X I Y) – probability "the moon is made of green cheese" is true given the evidence "there are mice on the moon" is true**

- » **P(X I ~ Y) – probability "the moon is made of green cheese" is true given the evidence "there are mice on the moon" is false**

# Probability with evidence example 1.4

◊ Let X be "the moon is made of green cheese"
Let Y be "there are mice on the moon"

» **P(X I Y) – probability "the moon is made of green cheese" is true given the evidence "there are mice on the moon" is true**

» **P(X I ~ Y) – probability "the moon is made of green cheese" is true given the evidence "there are mice on the moon" is false**

» **P(Y I X) – probability "there are mice on the moon" is true given the evidence "the moon is made of green cheese" is true**

# Probability with evidence example 2.1

◊ Let X be "you have cancer"
Let Y be "your cancer test was negative"
Let Z be "your parents had cancer"

# Probability with evidence example 2.2

◊ Let X be “you have cancer”
Let Y be “your cancer test was negative”
Let Z be  “your parents had cancer”

» **P(X | Y, Z) – probability “you have cancer” is true given “your cancer test was negative” and “your parents had cancer” are both true**

# Probability with evidence example 2.3

◊ Let X be "you have cancer"
Let Y be "your cancer test was negative"
Let Z be "your parents had cancer"

- » **P(X l Y, Z) – probability "you have cancer" is true given "your cancer test was negative" and "your parents had cancer" are both true**
- » **P(X l ~ Y, Z) – probability "you have cancer" is true given "your cancer test was negative" is false and "your parents had cancer" is true**

# Probability with evidence example 2.4

◊ Let X be "you have cancer"
Let Y be "your cancer test was negative"
Let Z be "your parents had cancer"

- » **P(X | Y, Z) – probability "you have cancer" is true given "your cancer test was negative" and "your parents had cancer" are both true**
- » **P(X | ~ Y, Z) – probability "you have cancer" is true given "your cancer test was negative" is false and "your parents had cancer" is true**
- » **P(Y, ~ Z | X) – probability "your cancer test was negative" is true and "your parents had cancer" is false given the evidence "you have cancer" is true**

# Burglary network questions

- P(alarm)?
- P(sensor)?
- P(alarm | burglary)?
- P(burglary | alarm)?
- P(burglary | alarm, ~ lightning)?
- P(alarm, ~ call | burglary)?