# Best-First Search Minimizing Space or Time 

## RBFS

Save space, take more time

## RBFS general properties

$\diamond$ Similar to A* algorithm developed for heuristic search

## RBFS general properties - 2

$\diamond$ Similar to A* algorithm developed for heuristic search
" Both are recursive in the same sense

## RBFS general properties - 3

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$\diamond$ Difference between $A^{*}$ and RBFS

## RBFS general properties - 3

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$\diamond$ Difference between $A^{*}$ and RBFS
" $\mathrm{A}^{*}$ keeps in memory all of the already generated nodes

## RBFS general properties - 4

$\diamond$ Similar to A* algorithm developed for heuristic search
" Both are recursive in the same sense
$\diamond$ Difference between A* and RBFS
" $\mathbf{A}^{*}$ keeps in memory all of the already generated nodes
" RBFS only keeps the current search path and the sibling nodes along the path

## RBFS space - 2

> When does RBFS suspend the search of a subtree?

## RBFS space - 3

» When does RBFS suspend the search of a subtree?
> When it no longer looks the best

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» When does RBFS suspend the search of a subtree?
> When it no longer looks the best
» What does it do when a subtree is suspended?

## RBFS space - 3

» When does RBFS suspend the search of a subtree?
> When it no longer looks the best
» What does it do when a subtree is suspended?
> It forgets the subtree to save space

## RBFS space - 4

» When does RBFS suspend the search of a subtree?
> When it no longer looks the best
» What does it do when a subtree is suspended?
> It forgets the subtree to save space
> What is the space complexity?

## RBFS space - 5

» When does RBFS suspend the search of a subtree?
> When it no longer looks the best
» What does it do when a subtree is suspended?
> It forgets the subtree to save space
> What is the space complexity?
> Linear the depth of the search

## RBFS space - 6

» When does RBFS suspend the search of a subtree?
> When it no longer looks the best
» What does it do when a subtree is suspended?
> It forgets the subtree to save space
> What is the space complexity?
> Linear the depth of the search

- Same as IDA*


## RBFS memory

» When RBFS suspends searching a subtree, what does it remember?

## RBFS memory - 2

» When RBFS suspends searching a subtree, what does it remember?
$>$ An updated f -value of the root of the subtree

## Updated f-values

> How does RBFS update the f-values?

## Updated f-values - 2

» How does RBFS update the f-values?
> Backing up the f-values in the same way as $A^{*}$ does

## f-value notation

$\diamond$ Static f-value
" $\mathrm{f}(\mathrm{N})$
$>$ Value returned by the evaluation function
> Always the same

## f-value notation - 2

$\diamond$ Static f-value
" $f(N)$
$>$ Value returned by the evaluation function
> Always the same
$\diamond$ Backed-up value
" $F(N)$
> Changes during the search

- Depends upon descendants of $\mathbf{N}$


## $F(N)$ definition

$\diamond$ RBFS backs up f-values in the same way as $A^{*}$
» How is $\mathrm{F}(\mathrm{N})$ defined?

## $\mathrm{F}(\mathrm{N})$ definition - 2

$\diamond$ RBFS backs up f-values in the same way as A*
» How is $\mathrm{F}(\mathrm{N})$ defined?
> If N has never been expanded?

## $F(N)$ definition - 3

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> How is $\mathrm{F}(\mathrm{N})$ defined?
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$-\mathbf{F}(\mathbf{N})=\mathbf{f}(\mathbf{N})$

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## $F(N)$ definition - 5

$\diamond$ RBFS backs up f-values in the same way as A*
» How is $\mathrm{F}(\mathrm{N})$ defined?
> If N has never been expanded?
$-\mathbf{F}(\mathbf{N})=\mathbf{f}(\mathbf{N})$
> If N has been expanded?
$-\mathbf{F}(\mathbf{N})=\min \left(\mathbf{F}\left(\mathbf{S}_{\mathbf{j}}\right)\right)$

- Where $S_{j}$ are the subtrees of $\mathbf{N}$


## Subtree exploration

» How does RBFS explore subtrees?

## Subtree exploration - 2

> How does RBFS explore subtrees?
> As in $A^{*}$, within a given f-bound

## Subtree exploration - 3

> How does RBFS explore subtrees?
> As in $A^{*}$, within a given f-bound
» How is the bound determined?

## RBFS subtree exploration - 4

> How does RBFS explore subtrees?
> As in $A^{*}$, within a given f-bound
» How is the bound determined?
> From the F-values of the siblings along the current search path

## Subtree exploration - 5

> How does RBFS explore subtrees?
> As in $A^{*}$, within a given f-bound
» How is the bound determined?
> From the F-values of the siblings along the current search path
> The smallest F-value

- The closest competitor


## Subtree exploration - 6

$\diamond$ Suppose $N$ is currently the best node

## Subtree exploration - 7

$\diamond$ Suppose N is currently the best node
$>\mathbf{N}$ is expanded

## Subtree exploration - 8

$\diamond$ Suppose N is currently the best node
$>\mathbf{N}$ is expanded
> N's children are expanded

## Subtree exploration - 9

$\diamond$ Suppose N is currently the best node
$>\mathrm{N}$ is expanded
> N's children are expanded
" Until when?

## Subtree exploration - 10

$\diamond$ Suppose N is currently the best node
$>\mathbf{N}$ is expanded
> N's children are expanded
> Until when?
$>\mathrm{F}(\mathrm{N})>$ Bound

## Subtree exploration - 10

$\diamond$ Suppose N is currently the best node
$>\mathbf{N}$ is expanded
> N's children are expanded

》 Until when?
$>F(N)>$ Bound
> Then what happens?

## Subtree exploration - 11

$\diamond$ Suppose N is currently the best node
$>\mathbf{N}$ is expanded
> N's children are expanded
» Until when?
$>F(N)>$ Bound
> Then what happens?
> Nodes below N are forgotten

## Subtree exploration - 12

$\diamond$ Suppose N is currently the best node
$>\mathbf{N}$ is expanded
> N's children are expanded
» Until when?
$>F(N)>$ Bound
» Then what happens?
> Nodes below N are forgotten
$>$ N's F-value is updated

## Subtree exploration - 13

$\diamond$ Suppose N is currently the best node
$>\mathbf{N}$ is expanded
> N's children are expanded
» Until when?
$>F(N)>$ Bound
» Then what happens?
> Nodes below N are forgotten
> N's F-value is updated
$>$ RBFS selects which node to expand next

## F-value inheritance

$\diamond$ F-values can be inherited from a node's parents

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$\diamond$ Let N be a node about to be expanded

## F-value inheritance - 3

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$\diamond$ Let N be a node about to be expanded
» If $F(N)>f(N)$ then $N$ had already been expanded

## F-value inheritance - 4

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$\diamond$ Let N be a node about to be expanded
» If $F(N)>f(N)$ then $N$ had already been expanded
» $\mathrm{F}(\mathrm{N})$ was determined from N's children

## F-value inheritance - 5

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> Children have been removed from memory

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$\diamond$ Suppose a child $\mathrm{N}_{\mathrm{k}}$ of N is generated again

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$\diamond$ Suppose a child $\mathrm{N}_{\mathrm{k}}$ of N is generated again
" Compute $f\left(\mathrm{~N}_{\mathrm{k}}\right)$

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» $\mathrm{F}(\mathrm{N})$ was determined from N's children
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$\diamond$ Suppose a child $\mathrm{N}_{\mathrm{k}}$ of N is generated again
" Compute $f\left(N_{k}\right)$
$\geqslant F\left(N_{k}\right)=\max \left(F(N), f\left(N_{k}\right)\right)$

## F-value inheritance - 9

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" Compute $f\left(N_{k}\right)$
" $F\left(N_{k}\right)=\max \left(F(N), f\left(N_{k}\right)\right)$
$>N_{k}$ 's F-value can be inherited from N

## F-value inheritance - 10

$\diamond$ F-values can be inherited from a node's parents
$\diamond$ Let N be a node about to be expanded
» If $F(N)>f(N)$ then $N$ had already been expanded
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- $\mathbf{N}_{k}$ was generated earlier


## F-value inheritance - 11

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"Compute $f\left(N_{k}\right)$
" $F\left(N_{k}\right)=\max \left(F(N), f\left(N_{k}\right)\right)$
$>N_{k}$ 's F-value can be inherited from $N$

- $\mathbf{N}_{\mathbf{k}}$ was generated earlier
$-F\left(N_{k}\right)$ was $\geq F(N)$, otherwise $F(N)$ would be smaller


## Fig 12.2 snapshots


$S$ is expanded
A is found to be the best child
$\mathrm{f}(\mathrm{n})$ in mocha $=\mathrm{g}(\mathrm{n})$ in clover $+\mathrm{h}(\mathrm{n})$ in magenta
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## Fig 12.2 snapshots - 2



A is expanded with bound 9
C has F-value 10
Stop expansion, backup F value
$f(n)$ in mocha $=g(n)$ in clover $+h(n)$ in magenta
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## Fig 12.2 snapshots - 3


$F=10$


Forget expansion from A
A has backed up F value 10
$E$ is best to expand next
$f(n)$ in mocha $=g(n)$ in clover $+h(n)$ in magenta
© Gunnar Gotshalks

## Fig 12.2 snapshots - 4


$F=10$

$E$ is expanded with bound 10
F has F-value 11

Stop expansion, backup F value
$\mathrm{f}(\mathrm{n})$ in mocha $=\mathrm{g}(\mathrm{n})$ in clover $+\mathrm{h}(\mathrm{n})$ in magenta
© Gunnar Gotshalks

## Fig 12.2 snapshots - 5



Forget expansion from $E$
E has backed up F value 11
A is best to expand next
$f(n)$ in mocha $=g(n)$ in clover $+h(n)$ in magenta
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## Fig 12.2 snapshots - 6



D has F-value 12

Stop expansion, backup F value
$f(n)$ in mocha $=g(n)$ in clover $+h(n)$ in magenta

## Fig 12.2 snapshots - 7



Forget expansion from A
A has backed up F value 12
$E$ is best to expand next
$\mathrm{f}(\mathrm{n})$ in mocha $=\mathrm{g}(\mathrm{n})$ in clover $+\mathrm{h}(\mathrm{n})$ in magenta
© Gunnar Gotshalks

## Fig 12.2 snapshots - 8


$\mathrm{f}(\mathrm{n})$ in mocha $=\mathrm{g}(\mathrm{n})$ in clover $+\mathrm{h}(\mathrm{n})$ in magenta © Gunnar Gotshalks

$E$ is expanded with bound 12
Reach goal, search ends

## Algorithm

function NewF (N, F(N), Bound)

```
    if F(N) > Bound then NewF := F(N)
```

else if goal( N ) then exit search with success
else if $\mathbf{N}$ has no children then NewF := infinity - dead end else for each child $\mathrm{N}_{\mathrm{k}}$ of N do
if $f(N)<F(N)$ then $F\left(N_{k}\right):=\max \left(F(N), f\left(N_{k}\right)\right)$
else $F\left(N_{k}\right):=f\left(N_{k}\right)$
sort children $\mathrm{N}_{\mathrm{k}}$ in increasing order of F-value while $F\left(N_{1}\right) \leq$ Bound and $F\left(N_{1}\right)<$ infinity do

Bound1 $:=\min \left(\right.$ Bound, $F$-value of sibling $N_{1}$ )
$F\left(N_{1}\right)$ := NewF ( $N_{1}, F\left(N_{1}\right)$, Bound1)
reorder nodes $N_{1}, N_{2}, \ldots$ according to new $F\left(N_{1}\right)$ end
end
NewF := $F\left(N_{1}\right)$

