Accumulators
More on Arithmetic
and
Recursion

listlen (L , N)

- L is a list of length N if ...

  listlen ([ ] , 0).

  listlen ([H | T] , N) :- listlen (T , N1) , N is N1 + 1.

  > On searching for the goal, the list is reduced to empty

  > On back substitution, once the goal is found, the counter is incremented from 0

- Following is an example sequence of goals (left hand column) and back substitution (right hand column)

listlen([a, b, c] , N).	N <== N1 + 1
listlen([b, c] , N1).	N1 <== N2 + 1
listlen([c] , N2).	N2 <== N3 + 1
listlen([] , N3).	N3 <== 0

Abstract the counter

◊ The following abstracts the counter part from listlen.

```
addUp ( 0 ).
addUp ( C )  :-  addUp ( C1 ) , C is C1 + 1.
```

◊ Notice the recursive definition occurs on a counter one smaller than in the head.

Count Up

◊ An alternate method is to count on the way to the fixed point value in the query

◊ The accumulator accumulates the result on the way to the goal.

```
adder ( C )  :-  adder ( 0 , C ).        Introduce accumulator
adder ( C , C )  :-  nl , write ( 'a ' ).
```

> The goal is reached when the accumulator reaches the fixed point count value

```
adder ( Acc1 , C )  :-  write ( 'b ' ) , Acc2 is Acc1 + 1
                                        , adder ( Acc2 , C ).
```

> The predicates in black always succeed, side effect is to write to the terminal – can see order of rule execution

listLen(L,N) – 2

◊ We can define list length using an accumulator

```
listln ( L , N )  :-  lenacc ( L , 0 , N ).
```

> Introduce the accumulator
> **Invariant:** length (L) + accumulator = N

```
lenacc ( [] , A , A ).

lenacc ( [ H | T ] , A , N )  :-  A1 is A + 1
                                , lenacc ( T , A1 , N ).
```

◊ Following is a sequence of goals

```
listln  ( [ a , b , c ] ,  N ).
lenacc ( [ a ,  b , c ] , 0 , N ).    N <== N1
lenacc ( [ b , c ] , 1 , N1 ).        N1 <== N2
lenacc ( [ c ] , 2 , N2 ).            N2 <== N3
lenacc ( [] , 3 , N3 ).               N3 <== 3
```

Sum a List of Numbers – no accumulator

- **sumList (List , Total)** asserts **List** is a list of numbers and **Total = + / List** .

```
sumList ( [ ] , 0 ).

sumList ( [ First | Rest ] , Total ) :-
    sumList ( Rest , Rest_total ) ,
    Total is First + Rest_total.
```

Sum a List of Numbers – with accumulator

◊ **sumList (List , Total)** asserts **List** is a list of numbers and **Total = + / List** .

» **Use an accumulator**

» **Here sumList asserts   Total = (+ / List) + Acc**

```
sumList ( List , Total ) :- sumList ( List , 0 , Total ).

sumList ( [ ] , Acc , Acc ).

sumList(  [ First l Rest ] , Acc , Total ) :-
    NewAcc is Acc + First ,
    sumList ( Rest , NewAcc , Total ).
```

A base case stops recursion

- A base case is one that stops recursion
  - » **This is a more general notion than the smallest problem.**
- Generate a sequence of integers from 0 to E, inclusive.
  - » **Need to stop recursion when we have reached E.**

```
numInRange ( N , E ) :- addUpToN ( 0 , N , E ).
addUpToE ( Acc , Acc , _ ).
addUpToE ( Acc , N , E ) :- Acc < E ,
                            Acc1 is Acc + 1 ,
                            addUpToE ( Acc1 , N , E ).
```

Base case, no recursion

Need guard to prevent selecting this rule to prevent recursion

Accumulator – Using vs Not Using

- The definition styles reflect two alternate definitions for counting
  - **Recursion – counts (accumulates) on back substitution.**
    - **Goal becomes smaller problem**
    - **Do not use accumulator**
  - **Iteration – counts up, accumulates on the way to the goal**
    - **Accumulate from nothing up to the goal**
    - **Goal "counter value" does not change**

- Some problems require an accumulator
  - **Parts explosion problem**
  - **Need intermediate results during accumulation**
    - **Partial sums of a list of numbers**

Factorial using recursion

- Following is a recursive definition of factorial

  Factorial (N) = N * Factorial (N – 1)

  factr (N , F) -- F is the factorial of N

  factr (0 , 1).

  factr (N , F) :- J is N – 1 , factr (J , F1)
  , F is N * F1.

- The problem **(J , F1)** is a smaller version of **(N , F)**
- Work toward the fixed point of a trivial problem
- Does not work for **factr (N ,120)** and **factr (N , F)**.
  - **Cannot do arithmetic J is N – 1 because N is undefined.**

Factorial using iteration – accumulators

- An iterative definition of factorial

```
facti ( N , F )  :-  facti ( 0 , 1 , N , F ).
facti ( N , F , N , F ).
facti ( I , Fi , N , F )  :-  invariant ( I , Fi , J , Fj )
                           ,  facti ( J , Fj , N , F ).

invariant ( I , Fi , J , Fj )  :-  J  is  I + 1 ,  Fj  is  J * Fi.
```

- The last two arguments are the goal and they remain the same throughout.
- The first two arguments are the accumulator and they start from a fixed point and accumulate the result
- Works for queries **facti (N ,120)** and **facti (N , F)** because values are always defined for the **is** operator.

Fibonacci – Ordinary Recursion

◊ Following is a recursive definition of the Fibonacci series. For reference here are the first few terms of the series

Index –	0	1	2	3	4	5	6	7	8	9	10	11	12
Value –	1	1	2	3	5	8	13	21	34	55	89	144	233

Fibonacci (N) = Fibonacci (N – 1)
+ Fibonacci (N – 2).

```
fib ( 0 , 1 ).
fib ( 1 , 1 ).
fib ( N , F )  :-  N1  is  N – 1 ,  N2  is  N – 2
                 ,  fib ( N1 , F1 ) , fib ( N2 , F2 )
                 ,  F  is  F1 + F2.
```

◊ Does not work for queries **fib (N , 8)** and **fib (N , F)**

» **Values for is operator are undefined.**

Fibonacci – Tail Recursion

◊ A tail recursive definition of the Fibonacci series

> **Tail recursion is equivalent to iteration**

```
fibt ( 0 , 1 ).
fibt ( 1 , 1 ).
fibt ( N , F )  :-  fibt ( 2 , 1 , 1 , N , F ).

fibt ( N , Last2 , Last1 , N , F )  :-  F  is  Last2 + Last1.
fibt ( I , Last2 , Last1 , N , F )  :-  J  is  I + 1
                                      ,  Fi  is  Last2 + Last1
                                      ,  fibt ( J , Last1 , Fi , N , F ).
```

◊ Works for queries **fibt (N , 120)** and **fibt (N , F)**

» **values are always defined for is operator.**

Compare Fib versions

◊ Compare ordinary recursion with tail recursion for the Fibonacci predicates.

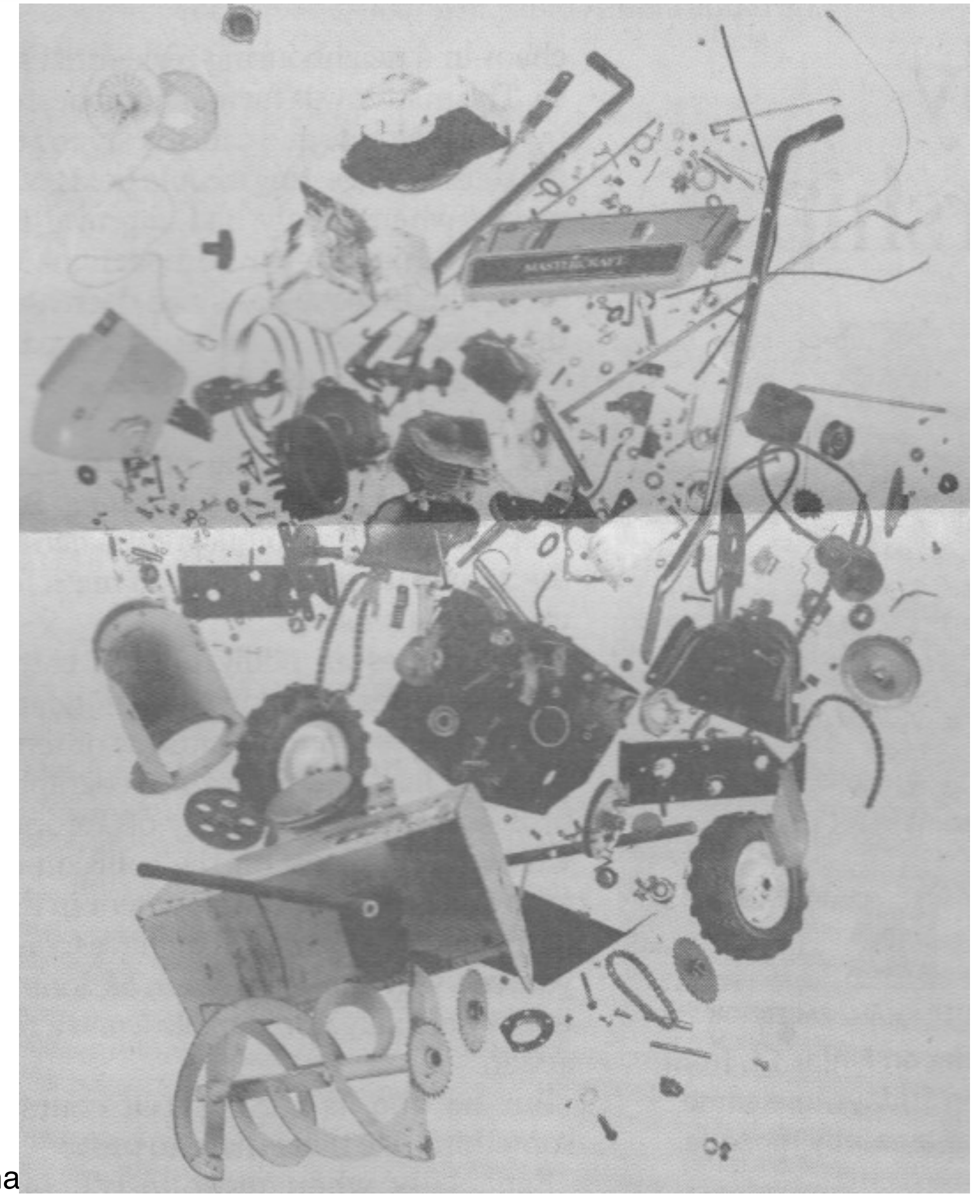
> **N is the n'th number in the series**
>
> **T1 is the time for tail recursion**
>
> **T2 is the time for ordinary recursion**

```
comparefib(N, T1, T2) :-
        statistics(runtime, _, _),
        fibt ( N , _ ),
        statistics(runtime, [_, T1]),
        fibr( N, _),
        statistics(runtime, [_,T2]).
```

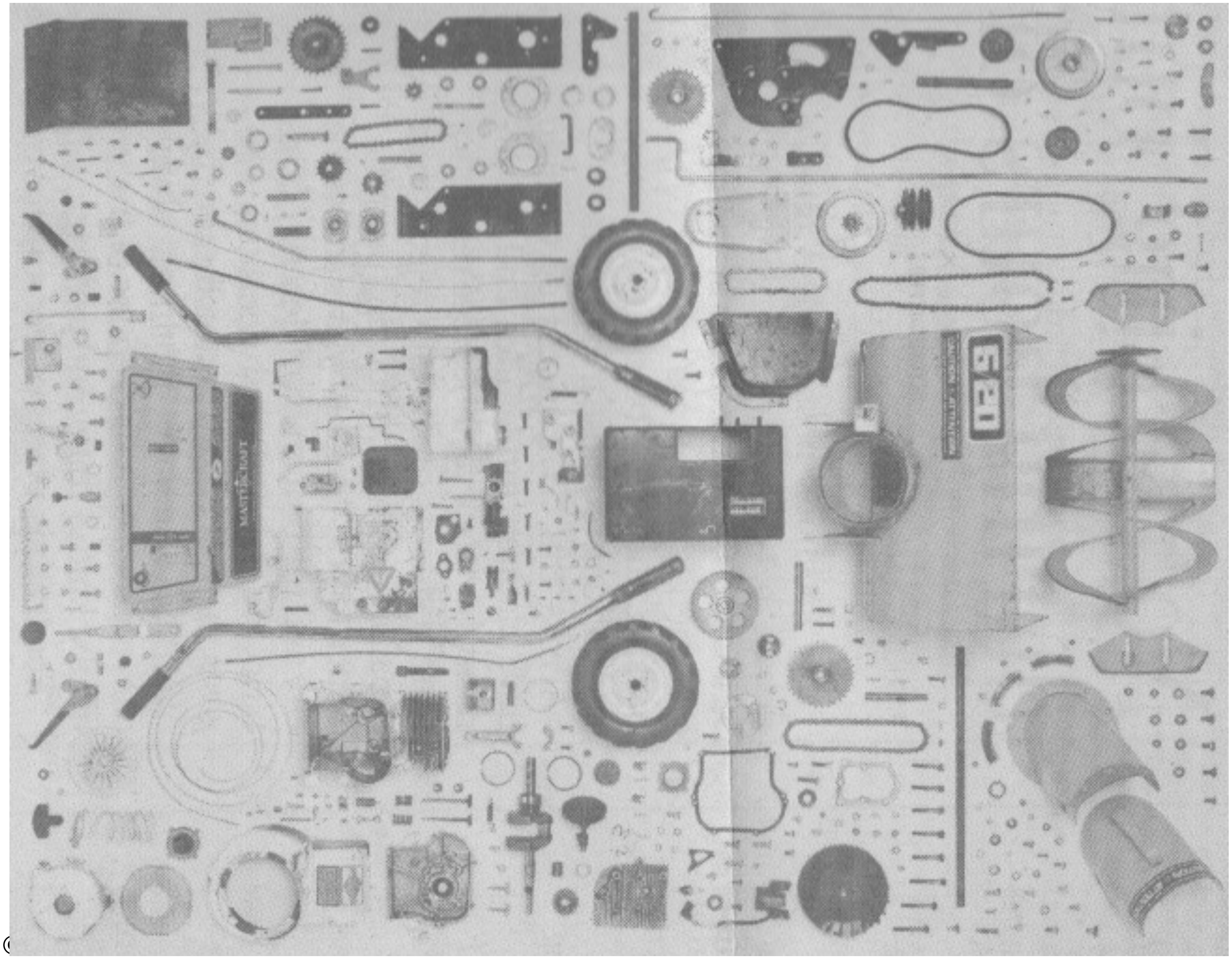
Parts Explosion – The Problem

◊ Parts explosion is the problem of accumulating all the parts for a product from a definition of the components of each part

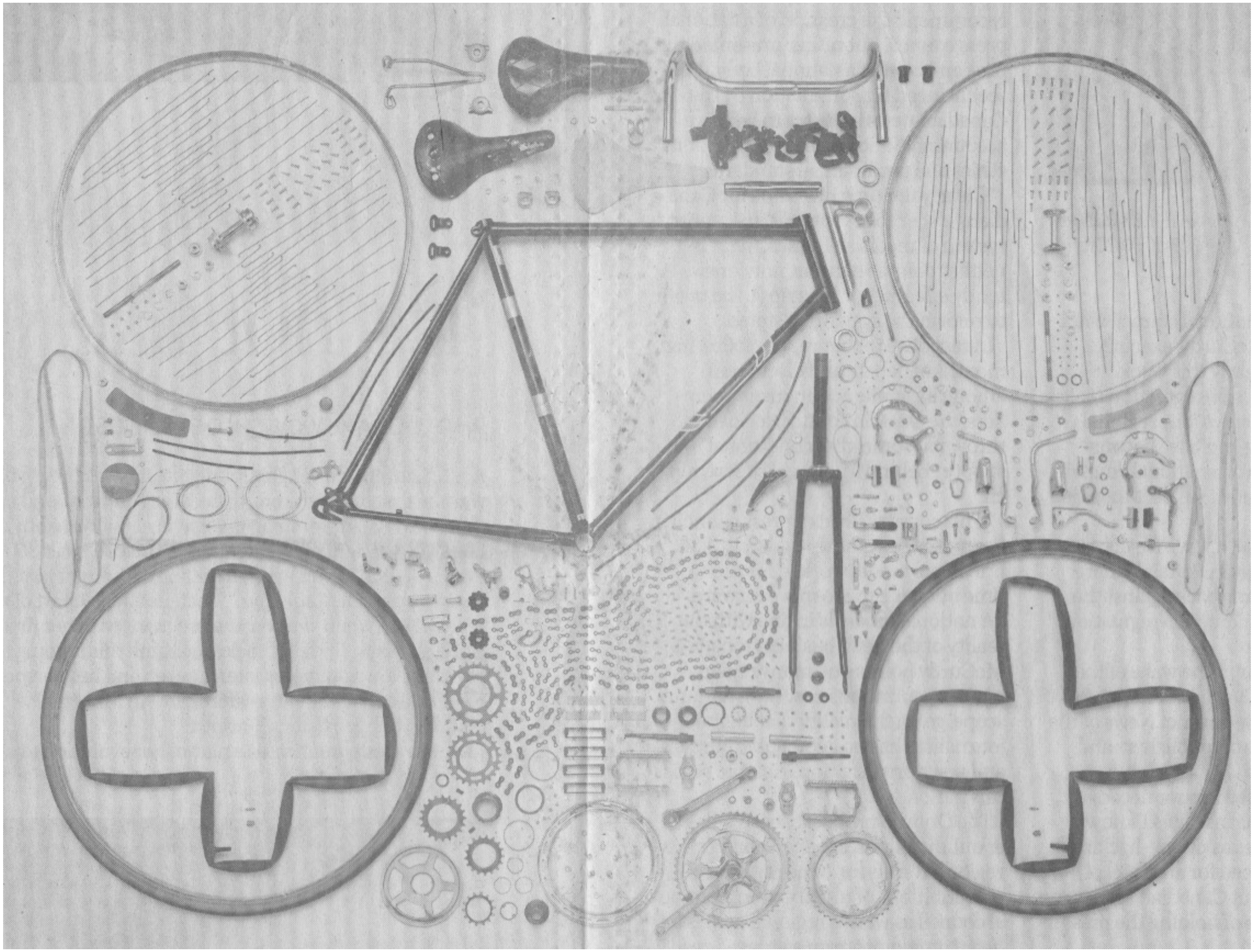
Snow Blower Parts View 1



Snow Blower Parts View 2



Bicycle Parts



Parts Explosion – Example

◊ Consider a bicycle we could have

> the following basic components

basicPart(spokes).   basicPart(rim).   basicPart(tire).
basicPart(inner_tube).   basicPart(handle_bar).
basicPart(front_ fork). basicPart(rear_fork).

> the following definitions for sub assemblies

assembly(bike, [wheel, wheel, frame]).
assembly(wheel, [spokes, rim, wheel_cushion]).
assembly(wheel_cushion, [inner_tube, tire]).
assembly(frame, [handle_bar, front_fork, rear_fork]).

Parts Explosion – The Problem 2

- ◊ We are interest in obtaining a parts list for a bicycle.

  [rear_ fork , front_ fork , handle_bar , tire , inner_tube , rim , spokes , tire , inner_tube , rim , spokes]

  > **We have two wheels so there are two tires, inner_tubes, rims and spokes.**

- ◊ Using accumulators we can avoid wasteful re-computation as in the case for the ordinary recursion definition of the Fibonacci series

Parts Explosion – Accumulator 1

- partsof (X ,P) – **P** is the list of parts for item **X**
- partsacc (X , A , P) – parts_of (**X**) || **A** = **P**.

  partsof (X , P) :- partsacc (X , [] , P).

|| is catenate (math append)

> **Basic part – parts list contains the part**

partsacc (X , A , [X | A]) :- basicPart (X).

> **Not a basic part – find the components of the part**

partsacc (X , A , P) :- assembly (X , Subparts) ,

> **parsacclist – parts_of (Subparts) || A = P**

partsacclist (Subparts , A , P).

Parts Explosion – Accumulator 2

- parsacclist (ListOfParts , AccParts , P)
  - parts_of (**ListOfParts**) || **AccParts** = **P**

    > **No parts $\Rightarrow$ no change in accumulator**

    partsacclist ([ ] , A , A).

    partsacclist ([Head | Tail] , A , Total) :-

    > **Get the parts for the first on the list**

    partsacc (Head , A , HeadParts)

    > **And catenate with the parts obtained from the rest of the ListOfParts**

    , partsacclist (Tail , HeadParts , Total).

Reverse a list with an accumulator

◊ Define the predicate **reverse (List , ReversedList)** that asserts **ReversedList** is the **List** in reverse order.

```
reverse ( List , Reversed ) :-
            reverse ( List , [ ] , Reversed ) .

reverse ( [ ] , Reversed , Reversed ) .

    reverse ( [ Head | Tail] ) || SoFar = Reversed

reverse ( [ Head | Tail ] , SoFar , Reversed ) :-
    reverse ( Tail , [ Head | SoFar ] , Reversed ) .
```

Reverse a list without accumulator

- Define the predicate **reverse (List , ReversedList)** that asserts **ReversedList** is the **List** in reverse order.

```
reverse ( [ ] , [] ) .

reverse ( [ Head | Tail ] , ReversedList ) :-
    reverse ( Tail , ReversedTail ) ,
    append ( ReversedTail , [ Head ] , ReversedList ) .
```

- Note the extra list traversal required by append compared to the accumulator version.

Difference Lists and Holes

- ◊ The accumulator in the parts explosion program is a stack
  - » **Items are stored in the reverse order in which they are found**
- ◊ How do we store accumulated items in the same order in which they are formed?
  - » **Use a queue**
- ◊ Difference lists with holes are equivalent to a queue

Examples for Holes

◊ Consider the following list

[a , b , c , d | X]

> **X is a variable indicating the tail of the list. It is like a hole that can be filled in once a value for X is obtained**

◊ For example

Res = [a , b , c , d | X] , X = [e , f].

> **Yields**

Res = [a , b , c , d , e , f]

Examples for Holes – 2

◊ Or could have the following with the hole going down the list

Res = [a , b , c , d | X]

> **more goal searching gives X = [e , f | Y]**
>
> **more goal searching gives Y = [h , i , j]**
>
> **Back substitution Yields**
>
> **Res = [a , b , c , d , e , f , h , i , j]**

Efficiency of List Concatenation

◊ Consider the definition of append

> **The concatenation of lists is inefficient when the first list is long**

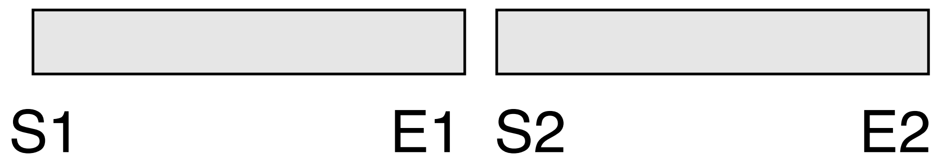
```
append ( [] , L , L ).

append ( [ X | L1 ] , L2 , [ X | L3 ] )
        :-  append ( L1 , L2 , L3 ) .
```

> **If we could skip the entire first part of the list in a single step, then concatenation of lists would be efficient**

Difference Lists Representation

- The list L
  - **L = [a, b, c]**
- Can be represented by two lists
  - **L1 = [a, b, c, d, e]** **L1 = [a, b, c]**
  - **L2 = [d, e]** **L2 = [ ]**
  - **L = L1 – L2** **L = L1 – L2**
- In fact L2 can be anything, so we can have the following
  - **L = [a, b, c, d, e | T] – [d, e | T]**
  - **L = [a, b, c | T] – T**
- The empty list [ ] = L – L, for any L

Difference Lists



(1) concat(S1 – E1 , S2 – E2 , S1 – E2) with E1 = S2

L1 = **[A , B , C]** = **[A , B , C | R1]** – **R1**
L2 = **[D , E]** = **[D , E | R2]** – **R2**

Pattern match (1) with (2)

(2) concat([A , B , C | R1] – R1 , [D , E | R2] – R2 , CL)
Using E1 = S2 we get

R1 = [D , E | R2]
CL = [A , B , C , D , E | R2] – R2

Parts Explosion – Difference List 1

- ◊ partsofd (X , P) – **P** is the list of parts for item **X**
- ◊ partsdiff (X , Hole , P) – parts_of (**X**) || **Hole** = **P**

  > **Hole and P are reversed compared to Clocksin & Mellish (v5) to better compare with accumulator version.**

```
partsofd ( X , P )  :-  partsdiff ( X , [ ] , P ).
```

  > **Base case we have a basic part, then the parts list contains the part**

```
partsdiff ( X , Hole , [ X | Hole ] )  :-  basicPart ( X ).
```

Parts Explosion – Difference List 2

> Not a base part, so we find the components of the part

partsdiff (X , Hole , P) :- assembly (X , Subparts)

> parsdifflistd – parts_of (Subparts) || Hole = P

, partsdifflist (Subparts , Hole , P).

Parts Explosion – Difference Lists 3

◊ parsdifflist (ListOfParts , Hole , P)
  – parts_of (**ListOfParts**) || **Hole** = **P**

partsdifflist ([ ] , Hole , Hole).

partsdifflist ([Head | Tail] , Hole , Total) :-

> **Get the parts for the first on the list**

partsdiff (Head , Hole1 , Total)

> **And catenate with the parts obtained from the rest of the ListOfParts**

, partsdifflist (Tail , Hole , Hole1).

Compare Accumulator with Hole

```
partsof   ( X , P ) :- partsacc  ( X , [ ] , P ).     Accumulator
partsofd ( X , P )  :- partsdiff ( X , [ ] , P ).     Difference/Hole

partsacc  ( X ,  A    , [ X | A ] )     :- basicPart ( X ).
partsdiff ( X , Hole , [ X | Hole ] )  :- basicPart ( X ).

partsacc  ( X ,  A    , P ) :- assembly (  X , Subparts )
                             , partsacclist ( Subparts , A    , P ).

partsdiff ( X , Hole , P ) :-  assembly (  X , Subparts )
                             , partsdifflist ( Subparts , Hole , P ).
```

Compare Accumulator with Hole – 2

```
partsacclist  ( [ ]  ,    A    ,  A    ).
partsdifflist (  [ ]  ,  Hole , Hole ).


partsacclist   ( [ Head | Tail ] , [A]    , Total )
                    :-  partsacc  ( Head ,  [A]    , HeadParts )
                      ,  partsacclist  ( Tail , HeadParts , Total ).

partsdifflist ( [ Head | Tail ] ,[Hole], Total )
                   :-  partsdiff ( Head ,  Hole1 ,     Total      )
                   ,   partsdifflist ( Tail  ,   [Hole]   ,  Hole1 ).
```