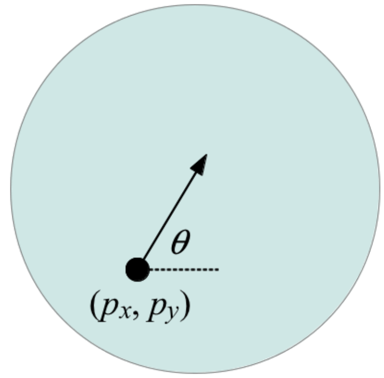
Suppose we have a turtle at position (p_x, p_y) pointed in the direction given by the angle θ as shown in the figure to the right.



The maximum distance that the turtle can move in a straight line can be found by solving for the intersection point of the line segment starting at (p_x, p_y) and pointing in the direction given by the angle θ. The parametric equation of the line using vector notation is given by:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \end{bmatrix} + t \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

Substituting into the equation of a circle we get:

$$\begin{aligned} x^2 + y^2 &= 1 \\ (p_x + t\cos(\theta))^2 + (p_y + t\sin(\theta))^2 &= 1 \\ p_x^2 + 2p_x t\cos(\theta) + t^2\cos^2(\theta) + p_y^2 + 2p_y t\sin(\theta) + t^2\sin^2(\theta) &= 1 \\ (\cos^2(\theta) + \sin^2(\theta))t^2 + 2(p_x\cos(\theta) + p_y\sin(\theta))t + (p_x^2 + p_y^2 - 1) &= 0 \\ at^2 + bt + c &= 0 \end{aligned}$$

where

$$\begin{aligned} a &= 1 && \text{because } \cos^2(\theta) + \sin^2(\theta) = 1 \\ b &= 2(p_x\cos(\theta) + p_y\sin(\theta)) \\ c &= p_x^2 + p_y^2 - 1 \end{aligned}$$

The roots of $at^2 + bt + c$ can be found using the quadratic equation. It turns out that the value of any real root is equal to the distance between (p_x, p_y) and the intersection point on the circle. There are several cases to consider for the values of the roots.

No real valued roots
In this case, (p_x, p_y) is outside of the circle and the direction of the line is such that there is no intersection with the circle. This should never occur because the turtle is supposed to remain inside of the circle.

Exactly one real valued root
In this case, the root must be $t = 0$, (p_x, p_y) is exactly on the perimeter of the circle, and the direction of the line is tangent to the circle; therefore, the maximum distance that the turtle can travel is $t = 0$.

Two roots greater than zero, or two roots less than zero
In this case, (p_x, p_y) is outside of the circle. This should never occur because the turtle is supposed to remain inside of the circle.

One root is zero and the other root is negative
The negative root corresponds to a point on the circle in the direction opposite to the direction that the turtle is pointed in; therefore, the maximum distance that the turtle can travel is $t = 0$.

One root is zero and the other root is positive
The positive root corresponds to a point on the circle in the direction that the turtle is pointed in; therefore, the maximum distance that the turtle can travel is the value of the positive root.

Two real valued roots with different signs
The positive root corresponds to a point on the circle in the direction that the turtle is pointed in; therefore, the maximum distance that the turtle can travel is the value of the positive root.

Finding the maximum distance
Based on the cases above, we can find the maximum distance that the turtle can move by doing the following:

1. Compute the coefficients a, b, and c
2. Find the largest real root t greater than or equal to zero
3. The maximum distance is given by t