MATH/EECS 1028: DISCRETE MATH FOR ENGINEERS
WINTER 2015
Tutorial 5 (Week of Feb 9, 2015)

Notes:

1. Assume $\mathbb{R}$ to denote the real numbers, $\mathbb{Z}$ to denote the set of integers $(\ldots, -2, -1, 0, 1, 2, \ldots)$ and $\mathbb{N}$ to denote the natural numbers $(1, 2, 3, \ldots)$.
2. Topics: Sequences, Logic.
3. Note to the TA: Attendance will be taken this week on Friday. Monday sections have a quiz this week.

Questions:

1. Predicates.

   Translate the following into English where $R(x)$ is "x is a comedian" and $H(x)$ is "x hops" and the domain consists of all animals. Then write down the negation of each statement in logic.

   (a) $\forall x(R(x) \rightarrow H(x))$

   (b) $\exists x(R(x) \wedge H(x))$

2. Express using logical operators, quantifiers and predicates: "The negation of a contradiction is a tautology".
3. Let $P(x), Q(x), R(x), S(x)$ be the statements "x is a baby", "x is logical", "x is able to manage a crocodile" and "x is despised" respectively. Suppose that the domain consists of all people. Express the following using quantifiers and the above predicates: "Nobody is despised who can manage a crocodile".
4. Nested quantifiers

   Express the following using predicates, quantifiers, logical connectives and mathematical operators where the domain is all integers.

   (a) "The sum of squares of two integers is greater than or equal to the square of their sum."

   (b) "The absolute value of the product of two integers equals the product of the abolute values of these integers."

   (c) "The difference of two negative integers is not necessarily negative."

   (d) "The absolute value of the sum of two integers does not exceed the sum of the abolute values of these integers."

(e) Express the negative of the following statement so that all negation symbols immediately precede predicates.

$\forall x \exists y (P(x,y) \to Q(x,y))$

5. Express the negative of the following statement so that all negation symbols immediately precede predicates.

   $\forall x \exists y \exists z (T(x,y,z) \vee Q(x,y))$

6. Let $F(x,y)$ be the statement "x can fool y", where the domain consists of all people in the world. Use quantifiers to express the following statements

   (a) "Everyone can be fooled by somebody".

   (b) Let $F(x,y)$ be the statement "x can fool y", where the domain consists of all people in the world. Use quantifiers to express the following statement: "There is no one who can fool everybody".

7. Express the following statement in predicate logic: "Every real number has exactly 2 square roots".

8. Express the negative of the following statement so that all negation symbols immediately precede predicates.

   $\forall x \exists y (P(x,y) \to Q(x,y))$

9. Inference.

   Determine if each of these statements is correct or incorrect and explain why.

   (a) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.

   (b) Quincy likes all action movies. Quincy likes the movie My Cousin Vinny. Therefore, My Cousin Vinny is an action movie (denying the hypothesis).

   (c) All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.

   (d) Every CSE major takes discrete math. Natasha is taking discrete math. Therefore, Natasha is a CSE major.

   (e) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.

   (f) Everyone who eats granola daily is healthy. Linda is not healthy. Therefore, Linda does not eat granola daily.

   (g) Express using logical operators, quantifiers and predicates: "The conjunction of two tautologies is a tautology".

   (h) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \to \neg S(x))$ and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

10. Proof by cases.

    Prove that $n, n+7$ cannot both be perfect cubes where n is an integer greater than 1.

11. Functions

    Define a function f from $\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ as $f(m,n) = m^2 - 10$. Is this function onto? Prove your answer.

12. Prove the following statement: If a, b, c are odd integers, then $ax^2 + bx + c = 0$ does not have a rational number solution.

13. Let $p < q$ be two consecutive odd primes. Prove that $p + q$ is a composite number, having at least three, not necessarily distinct, prime factors.

14. A function $f(x)$ is said to be strictly increasing if $f(b) > f(a)$ for all $b > a$. Prove that a strictly increasing function from $\mathbb{R}$ to itself is one-to-one.