MATH/EECS 1028: Discrete Math for Engineers
Winter 2015
Tutorial 2 (Week of Jan 19, 2015)

Notes:

1. Assume $\mathbb{R}$ to denote the real numbers, $\mathbb{Z}$ to denote the set of integers $(\ldots, -2, -1, 0, 1, 2, \ldots)$ and $\mathbb{N}$ to denote the natural numbers $(1, 2, 3, \ldots)$.
2. Topics: Number systems, sets, set operations, functions.
3. Note to the TA: Attendance will be taken each week. There is a short quiz this week containing a couple of questions that are very similar to the ones done last week.

Questions:

1. Can you conclude that $A = B$ if A, B, C are sets such that
   (a) $A \cap C = B \cap C$?
   (b) $A \cup C = B \cup C$ and $A \cap C = B \cap C$?
2. Let $A_i = \{\ldots, -2, -1, 0, 1, \ldots, i\}$. Find
   (a) $\cup_{i=1}^{n} A_i$?
   (b) $\cap_{i=1}^{n} A_i$?
3. Find $\cup_{i=1}^{\infty} A_i$ and $\cap_{i=1}^{\infty} A_i$ if $i \in \mathbb{N}$ and
   (a) $A_i = \{x | x \in \mathbb{N}, x \geq i\}$
   (b) $A_i = \{x | x \in \mathbb{R}, x > i\}$
4. The **successor** of the set A is the set $A \cup \{A\}$. Find the successor of the sets
   (a) $\emptyset$
   (b) $\{\emptyset\}$
   (c) $\emptyset \in \{\emptyset\}$
   (d) $\{\emptyset, \{\emptyset\}\}$
5. Show that if A, B are sets then $\overline{A \cup B} = \overline{A} \cap \overline{B}$, by showing that each side is a subset of the other side.
6. Determine whether f is a function from $\mathbb{Z}$ to $\mathbb{R}$ if
   (a) $f(n) = \sqrt{n^2 + 1}$?
   (b) $f(n) = \frac{1}{n^2 - 4}$?
7. Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto (surjective) if

(a) $f(m,n) = m+n+1$?

(b) $f(m,n) = |m| - |n|$?

(c) $f(m,n) = m^2 - 4$?

8. Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$

   (a) $f(x) = -3x^2 + 7$

   (b) $f(x) = \frac{x+1}{x+2}$.

9. Let $f(x) = 2x$ where the domain of f is $\mathbb{R}$. What is

   (a) $f(\mathbb{Z})$?

   (b) $f(\mathbb{N})$?

   (c) $f(\mathbb{R})$?

10. Suppose that f is an invertible function from Y to Z and g is an invertible function from X to Y. Show that the inverse of the composition $f \circ g$ is given by $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

11. Find $g^{-1}(3)$ given $g(x) = \frac{3x+1}{2x+g(x)}$.

12. Let f be the function $f(x) = ax^2 - \sqrt{2}$ for some positive real number a.
    If $f(f(\sqrt{2})) = -\sqrt{2}$ what is a?

13. Let $x = 2^{\log_b 3}$ and $y = 3^{\log_b 2}$. Find $x - y$.

14. If $\frac{\log_b a}{\log_c a} = \frac{19}{99}$ and $\frac{b}{c} = c^k$, compute k.

15. Try this for practice: Let f be a function from A to B. Let S, T be subsets of B. Show that $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$.