MATH/EECS 1028: Discrete Math for Engineers
Winter 2015
Tutorial 1 (Week of Jan 12, 2015)

Notes:

1. Assume $\mathbb{R}$ to denote the real numbers, $\mathbb{Z}$ to denote the set of integers $(\ldots, -2, -1, 0, 1, 2, \ldots)$ and $\mathbb{N}$ to denote the natural numbers $(1, 2, 3, \ldots)$.
2. Topics: Number systems, quadratic equations, exponents, sets, set operations.
3. Attendance will be taken each week. There is nothing to be submitted this week.
4. In the tutorials next week they have to submit a couple of questions that are very similar to the ones done this week.

Questions:

1. Simplify: $5 \times (3^{2x} - 9^x \times 5^{-1}) \times 2^{-2}$

   Solution:

$$\begin{aligned} 5 \times (3^{2x} - 9^x \times 5^{-1}) \times 2^{-2} &= 5 \times (9^x - 9^x \times 5^{-1}) \times 1/4 \\ &= 5 \times 4/5 \times 9^x \times 1/4 \\ &= 9^x \end{aligned}$$

2. Solve the equation: $9^x - 5 \times 3^x + 4 = 0$.

   Solution: Let $t = 3^x$. Then we have the quadratic equation: $t^2 - 5t + 4 = 0$. We can solve this equation to get $t = 4, 1$ which correspond to $x = \log_3 4, 0$.

3. Solve the equation: $9^{x^2} = 27^{3x-3}$.

   Solution: The left hand side equals 3^{2x^2}. The right hand side equals 3^{9x-9}. Taking logs base 3 of both sides we get $2x^2 = 9x - 9$ or $2x^2 - 9x + 9 = 0$, which is a quadratic equation that has solutions $x = 3/2$ and $x = 3$.

4. Are these statements correct? Why or why not?

   (a) $\{\{1\}\} = \{1\}$
   Solution: No. The right hand side contains one number, the left hand side contains one set.

   (b) $\emptyset \neq \{\emptyset\}$
   Solution: Yes. The right hand side contains one element, the left hand side is the empty set.

   (c) $\emptyset \in \{\emptyset\}$
   Solution: Yes. The right hand side contains $\emptyset$ as an element.

(d) $\emptyset \in \{\{\emptyset\}\}$

Solution: No. The right hand side contains a set but not the element $\emptyset$.

(e) $\emptyset \subseteq \emptyset$

Solution: Yes. The empty set is a subset of every set.

(f) $\emptyset \subseteq \{\emptyset\}$

Solution: Yes. The empty set is a subset of every set.

(g) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

Solution: Yes. The right hand side contains the left hand side and another element.

5. Enumerate the following sets

(a) The power set of $\{\emptyset, \{a\}\}$

Solution: $\{\emptyset, \{\emptyset\}, \{\{a\}\}, \{\emptyset, \{a\}\}\}$.

(b) A^3 when $A = \{a\}$.

Solution: $\{(a, a, a)\}$.

(c) $A \times B \times C, (A \times B) \times C$ when $A = \{a\}, B = \{b\}, C = \{c\}$.

Solution: $A \times B \times C = \{(a, b, c)\}$, $(A \times B) \times C = \{((a, b), c)\}$.

6. Show that

(a) If A is a set $A \times \emptyset = \emptyset \times A = \emptyset$.

Solution: Using the definition of Cartesian product, each ordered pair $(x, y) \in A \times \emptyset$ satisfies $y \in \emptyset$. Since there are no such y, $A \times \emptyset = \emptyset$. Using identical arguments one can show $\emptyset \times A = \emptyset$.

(b) If A, B, C, D are sets and $A \subseteq B, C \subseteq D$ then $A \times C \subseteq B \times D$.

Solution: Consider any element (ordered pair) $(x, y) \in A \times C$. We know that $x \in A, y \in C$. Since $A \subseteq B, C \subseteq D$, so it follows that $x \in B, y \in D$. Therefore $(x, y) \in B \times D$.

Since every element of $A \times C$ belongs to $B \times D$, it follows that $A \times C \subseteq B \times D$.

7. (a) Suppose that A, B are sets and $A \times B = \emptyset$. What can you conclude about A, B?

Solution: Using the reasoning of Q6(a) above, we conclude that either $A = \emptyset$ or $B = \emptyset$.

Note that this allows for both A, B to be empty.

(b) What is the cardinality of $\{\{a, b\}, \{c, d\}\}$?

Solution: 2.

8. Prove that $\sqrt{3}$ is not rational.

Solution: We replicate the proof of the irrationality of $\sqrt{2}$ shown in class.

Let us assume it is rational. So we can assume that $\sqrt{3} = m/n$ for integers m, n and $n \neq 0$. Assume further that m, n have no common factors greater than 1. Then we have

$$
\begin{aligned}
\sqrt{3} &= m/n \text{ Squaring both sides, we get} \\
3 &= m^2/n^2 \\
3n^2 &= m^2 \text{ Since the left side is a multiple of 3, so is the right side. So} \\
m &= 3k \text{ for some integer } k. \text{ So} \\
3n^2 &= 9k^2 \\
n^2 &= 3k^2 \text{ Since the right side is a multiple of 3, so is the left side. So} \\
n &= 3j \text{ for some integer } j.
\end{aligned}
$$

We just showed that m, n have a common factor of 3. This is a contradiction since we assumed m, n have no common factors. Therefore $\sqrt{3}$ cannot be rational.

9. Draw the Venn diagram for the set $A \cap (B - C)$

   Solution: Omitted.

10. What can you say about the sets A, B if

    (a) $A \cup B = A$
    Solution: $B \subseteq A$.

    (b) $A \cap B = A$
    Solution: $A \subseteq B$.

    (c) $A \cup B = B \cup A$
    Solution: Always true for any A, B.

    (d) $A - B = A$
    Solution: $A \cap B = \emptyset$.

    (e) $A - B = B - A$
    Solution: $A = B$.