Next: Predicate Logic

Ch 1.4

– Predicates and quantifiers
– Rules of Inference

Predicate Logic

- A predicate is a proposition that is a function of one or more variables.

  E.g.: P(x): x is an even number. So P(1) is false, P(2) is true,….
- Examples of predicates:
  – Domain ASCII characters - IsAlpha(x) : TRUE iff x is an alphabetical character.
  – Domain floating point numbers - IsInt(x): TRUE iff x is an integer.
  – Domain integers: Prime(x) - TRUE if x is prime, FALSE otherwise.

Quantifiers

- describes the values of a variable that make the predicate true. E.g. $\exists x\ P(x)$
- Domain or universe: set of values taken by a variable (sometimes implicit)

Two Popular Quantifiers

- Universal: $\forall x\ P(x)$ – "P(x) for all x in the domain"
- Existential: $\exists x\ P(x)$ – "P(x) for some x in the domain" or "there exists x such that P(x) is TRUE".
- Either is meaningless if the domain is not known/specified.
- Examples (domain real numbers)
  - $\forall x\ (x^2 >= 0)$
  - $\exists x\ (x > 1)$
  - $(\forall x > 1)\ (x^2 > x)$ – quantifier with restricted domain

Using Quantifiers

Domain integers:

- Using implications: The cube of all negative integers is negative.

  $\forall x\ (x < 0) \rightarrow (x^3 < 0)$

- Expressing sums :

  $$\forall n\ (\sum_{i=1}^{n} i = n(n+1)/2)$$

Aside: summation notation

Scope of Quantifiers

- $\forall$ $\exists$ have higher precedence than operators from Propositional Logic; so $\forall x\, P(x) \lor Q(x)$ is not logically equivalent to $\forall x\, (P(x) \lor Q(x))$
- $\exists\, x\, (P(x) \land Q(x)) \lor \forall x\, R(x)$

  Say P(x): x is odd, Q(x): x is divisible by 3, R(x): $(x=0) \lor (2x > x)$
- Logical Equivalence: $P \equiv Q$ iff they have same truth value no matter which domain is used and no matter which predicates are assigned to predicate variables.