Subsets

- $A \subseteq B$: $\forall x\, (x \in A \rightarrow x \in B)$

  Theorem: For any set S, $\phi \subseteq S$ and $S \subseteq S$.
- Proper subset: $A \subset B$: $\forall x\, (x \in A \rightarrow x \in B) \wedge \exists\, x\, (x \in B \wedge x \notin A)$
- Power set P(S) : set of all subsets of S.
- P(S) includes S, ϕ.
- Tricky question – What is $P(\phi)$?

$P(\phi) = \{\phi\}$

Similarly, $P(\{\phi\}) = \{\phi, \{\phi\}\}$

Set operations

- Union – $A \cup B = \{ x \mid (x \in A) \vee (x \in B)\}$
- Intersection - $A \cap B = \{ x \mid (x \in A) \wedge (x \in B)\}$

  Disjoint sets - A, B are disjoint iff $A \cap B = \phi$
- Difference – $A - B = \{x \mid (x \in A) \wedge (x \notin B)\}$

  Symmetric difference
- Complement – A^c or $\bar{A} = \{x \mid x \notin A\} = U - A$
- Venn diagrams

Laws of set operations

- Page 130 – Remember De Morgan's Laws and distributive laws.
- Proofs can be done with Venn diagrams.

  E.g.: $(A \cap B)^c = A^c \cup B^c$

Proofs via membership tables (page 131)

Introduction to functions

A function from A to B is an assignment of exactly one element of B to each element of A.

E.g.:

- Let A = B = integers, $f(x) = x+10$
- Let A = B = integers, $f(x) = x^2$

Not a function

- A = B = real numbers $f(x) = \sqrt{x}$
- A = B = real numbers, $f(x) = 1/x$

Terminology

- A = Domain, B = Co-domain
- f: $A \rightarrow B$ (not "implies")
- range(f) = $\{y \mid \exists\, x \in A\ f(x) = y\} \subseteq B$
- int floor (float real){ … }
- $f_1 + f_2$, $f_1 f_2$
- One-to-one INJECTIVE
- Onto SURJECTIVE
- One-to-one correspondence BIJECTIVE

Proving injection (pg 142)

Property : A function is injective if and only if $f(a) \neq f(b)$ whenever $a \neq b$

Proving injection - Show that if $f(a) = f(b)$ then $a=b$

Example: $f(x) = 2x+1$, $f(x) = x^3$

Proving surjection (pg 143)

Property: If $f:A \rightarrow B$, for any $b \in B$, there must exist $a \in B$, $f(a) = b$

Proving surjection - consider an arbitrary $b \in B$. Find a such that $f(a) = b$.

Example: $f(x) = 2x+1$, $f(x) = x^3$

Bijective functions (pg 144)

- Both injective and surjective
- A bijective function is invertible
- Inverse: $f^{-1}(y) = x$ iff $f(x) = y$
- $f^{-1}(x) \neq 1/f(x)$

Operations with functions

- Addition:

  Example: $f(x) = 2x$, $g(x) = x^2 + 1$

- Composition: If $f: A \to B$, $g: C \to A$, then $f \circ g: C \to B$, $f \circ g(x) = f(g(x))$

  Example: $f(x) = 2x$, $g(x) = x^2 + 1$