MATH/EECS 1028: Discrete Math for Engineers
Winter 2015
Assignment 2 (Released Mar 18, 2015)
Submission deadline: 1:15 pm, April 1, 2015

Notes:

1. The assignment can be handwritten or typed. It MUST be legible.
2. You must do this assignment individually.
3. Submit this assignment only if you have read and understood the policy on academic honesty on the course web page. If you have questions or concerns, please contact the instructor.
4. Use the dropbox near the EECS main office to submit your assignments, OR submit your assignment in the first TEN minutes of class on the day of the deadline. No late submissions will be accepted. Please do not send files by email.
5. Your answers should be precise and concise. Points may be deducted for long, rambling arguments.
6. Assume $\mathbb{R}$ to denote the real numbers, $\mathbb{Z}$ to denote the set of integers $(\ldots, -2, -1, 0, 1, 2, \ldots)$ and $\mathbb{N}$ to denote the natural numbers $(1, 2, 3, \ldots)$.

Question 0

[4 points] If you did not submit Q4 of the last assignment include it here. If you did, you need not submit it again.

Question 1

[4 points] Your bank requires 4 digit strings as your password for transactions at the automated banking machines. They also insist that your string should not have three or more consecutive digits in increasing order. Thus 1234 and 7234 are not considered valid. How many valid strings are there? Show how you arrived at your answer.

Question 2

[4 points] Five different circles are drawn on a sheet of paper. What is the maximum possible number of intersections of these circles? Why?

Question 3

[4 points] How many natural numbers less than 1000 are there in which the digits are strictly increasing from left to right?

Question 4

[4 points] Consider only lowercase english characters for this question. How many strings (not necessarily valid English words) of length 5 are there that contain at least 2 consecutive letters that are the same? Justify your answer.

Question 5

[4 points] A very common application of strong induction is in solving recursively defined sequences. A recursively defined sequence is one in which each term (except the first few) are defined in terms of previous terms. Some such sequences are easy to solve. For example, the definition $a_1 = 1$ and for $n > 1$, $a_n = a_{n-1}+1$ is easily seen to have the solution $a_i = i, i \in \mathbb{N}$.

Consider the recursively defined sequence $a_1 = 5, a_2 = 10$ and for $n > 2$, $a_n = 2a_{n-1} + a_{n-2}$. Prove using strong induction that for $n \geq 3$, $a_n < 3^n$.