Test 1

First Name: ____________________

Last Name: ____________________

Student Number: ____________________

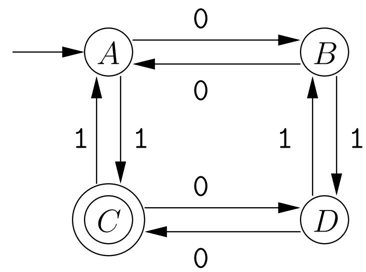
This test lasts 80 minutes. *No aids allowed.*
You may use any result that was proved in class or in the textbook without reproving it.
Make sure your test has 5 pages, including this cover page.
Answer in the space provided. (If you need more space, use the reverse side of the page and indicate ***clearly*** *which part of your work should be marked.)*
Write legibly.

Question 1	/4
Question 2	/4
Question 3	/3
Question 4	/3
Question 5	/3
Question 6	/3
Total	/20

1. [4 marks] Let $L_1 = \{x \in \{\mathtt{0}, \mathtt{1}\}^* : x \text{ contains a } \mathtt{1} \text{ and the length of } x \text{ is a multiple of } 3\}$. Draw the transition diagram of a deterministic finite automaton for L_1. For each state of your machine, describe the set of strings that take the machine to that state.

2. [4 marks] Consider the DFA M shown at right. Recall that $\delta^*(A, x)$ denotes the state that M is in after processing the string x, starting from the initial state A. Give a careful proof of the following claim.

Claim: For any string $x \in \{0, 1\}^*$,
$\delta^*(A, x) \in \{C, D\}$ if x contains an odd number of 1's, and
$\delta^*(A, x) \in \{A, B\}$ if x contains an even number of 1's.



3. [3 marks] Let $L_3 = \{x \in \{\texttt{0},\texttt{1}\}^* : x \text{ starts with } \texttt{0} \text{ and has even length}\}$.
Write a regular expression for L_3.

4. [3 marks] Let $L_4 = \{x \in \{\texttt{0},\texttt{1}\}^* : x \text{ contains exactly twice as many } \texttt{0}\text{'s as } \texttt{1}\text{'s}\}$. For example, `001010` in in L_4 but `11000` is not in L_4. Prove that L_4 is not regular.

5. [3 marks] Give a high-level description of how a non-deterministic finite automaton N can be transformed into a deterministic finite automaton M that accepts the same language. Your answer must fit inside the box below. Anything written outside the box will be ignored.

6. [3 marks] If $L \subseteq \Sigma^*$ is a language, define $PREFIX(L)$ to be the set of all strings that are prefixes of strings in L. More formally,

$$PREFIX(L) = \{x \in \Sigma^* : \text{there exists } y \in \Sigma^* \text{ such that } xy \in L\}.$$

Show that for every regular language L, the language $PREFIX(L)$ is also regular.