

**Homework Assignment #8**  
**Due: July 29, 2015 at 7:00 p.m.**

1. Prove the following statements.

(a) A set  $A$  is countable if and only if there is a one-to-one function  $f : A \rightarrow \mathbb{N}$ .

(b) If  $A$  and  $B$  are countable sets, then  $A \cup B$  is countable.

(c) For all positive integers  $n$ , if  $A_1, \dots, A_n$  are countable sets then  $\bigcup_{i=1}^n A_i$  (the union of all of the  $A_i$ 's) is countable.

2. Is the following statement true? For all languages  $L$  and  $L'$ , if  $L' \subseteq L$  and  $L$  is recognizable, then  $L'$  is recognizable. Prove your answer is correct.