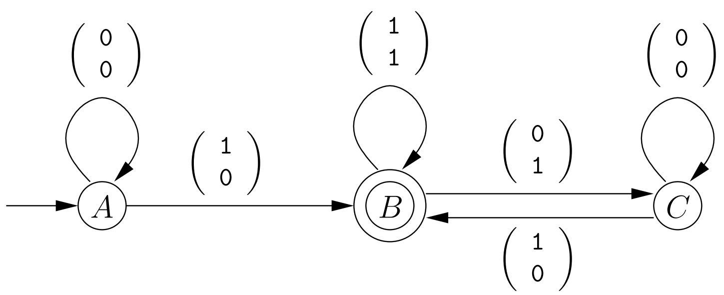
York University | EECS 2001 | June 1, 2015

Homework Assignment #3
Due: June 10, 2015 at 7:00 p.m.

1. In this question, we shall consider a finite automaton that uses the input alphabet $\Sigma = \left\{ \binom{0}{0}, \binom{0}{1}, \binom{1}{0}, \binom{1}{1} \right\}$. If $x \in \Sigma^*$ is a string, we define $top(x)$ and $bottom(x)$ to be the two numbers represented (in binary) by the top row and the bottom row of bits in x. For example, if $x = \binom{1}{0}\binom{0}{1}\binom{1}{1}\binom{0}{1}\binom{1}{0}\binom{1}{1}$, then $top(x) = 43$ (since the top row of x is `101011`, which is the binary representation of 43) and $bottom(x) = 29$ (since the bottom row of x is `011101`, which is the binary representation of 29).

Now, consider the finite automaton shown below. We use the convention that if no transition is shown, the automaton moves to the reject state (not shown) and then stays there forever.



(a) Find four different strings that the finite automaton accepts. For each string x that you find, write down $top(x)$ and $bottom(x)$. At least one of the strings you find should include the character $\binom{0}{1}$.

(b) Fill in the blanks in the following claim with simple statements about the relationship between $top(x)$ and $bottom(x)$.

Claim: For all strings $x \in \Sigma^*$ of length at least 1:

(i) $\delta^*(A, x) = A$ if and only if $top(x) = bottom(x) = 0$.
(ii) $\delta^*(A, x) = B$ if and only if ________________________
(iii) $\delta^*(A, x) = C$ if and only if ________________________

(c) Give a detailed proof of the "only if" part of all three claims in part (b).

Note: it is important that you prove the correct direction. If you prove the "if" direction, you will not get credit for this question.

(d) Complete the following claim with a simple statement about x. For all $x \in \Sigma^*$ of length at least one, the finite automaton accepts x if and only if ________________________. Indicate why your answer follows from your claim in part (b).