

**Homework Assignment #11**  
**Due: August 19, 2015 at 7:00 p.m.**

1. Let  $L_3 = \{0^i 1^j 2^k : i = j \text{ or } j > 2k\}$ .
  - (a) Give a context-free grammar for  $L_3$ . You do not have to prove your answer is correct. However, for each variable you use in your grammar, you must state the set of strings that is generated by that variable.
  - (b) Use a proof by induction to show that for all  $i \geq 0$ , your grammar generates  $0^i 1^i$ .
  
2. Consider the following context-free grammar.

$$S \rightarrow S1S \mid S2S \mid 0$$

- (a) Complete the following sentence by specifying properties of string  $x$ : A string  $x \in \{0, 1, 2\}^*$  is generated by the grammar if and only if \_\_\_\_\_.  
You do not have to prove your answer is correct.
  - (b) Draw a finite automaton that accepts a string if and only if it is generated by the grammar. You do not have to prove your answer is correct.
3. Consider the following context-free grammar with starting symbol  $S$ .

$$\begin{aligned} S &\rightarrow aSb \mid Z \\ Z &\rightarrow bY \mid Ya \\ Y &\rightarrow bY \mid aY \mid \varepsilon \end{aligned}$$

Let  $E = \{a^n b^n : n \geq 0\}$ . You may use the following (simple) lemma without proving it.

**Lemma:** Every string generated by the variable  $Z$  is not in the set  $E$ .

Give a careful proof of the following claim.

**Claim:** Every string generated by the grammar is *not* in the set  $E$ .