

## Homework Assignment #1

### Due: May 27, 2015 at 7:00 p.m.

**When you submit your solution to Assignment 1, you should also hand in the declaration on academic honesty. Without this declaration, your assignment will not be marked. Please do not staple the declaration to your solutions.**

1. Willemina wrote the following algorithm, which returns a boolean value (TRUE or FALSE). She wants you to help her understand exactly what it does and why it works. In the following pseudocode,  $X \leftarrow expr$  is used to assign the value of the expression  $expr$  to the variable  $X$ . Assume that the input parameter  $S$  is a string over the alphabet  $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ . If  $S$  is a string of length  $\ell$ , then the individual characters of  $S$  are  $S(1), S(2), S(3), \dots, S(\ell)$ .

```

1  MYSTERY(string  $S$ ) : boolean
2      int  $\ell \leftarrow length(S)$ 
3      int  $i \leftarrow 1$ 
4      int  $x \leftarrow 0$ 
5      while  $i \leq \ell$ 
6          if  $S(i) = \mathbf{a}$  then
7               $x \leftarrow x + 1$ 
8          end if
9           $i \leftarrow i + 1$ 
10     end while
11     return  $(x \bmod 2 = 1)$       //  $x \bmod 2$  is the remainder when  $x$  is divided by 2
12 end MYSTERY
```

- (a) Fill in the following blank with a simple, English statement about  $S$  that makes the sentence true:  
 MYSTERY( $S$ ) returns TRUE if and only if \_\_\_\_\_.
- (b) Which one of the following statements is true *every* time line 5 is executed (for *every* possible input string  $S$ )?
- (1)  $i \geq 1$  and the number of **a**'s among the first  $i$  characters of  $S$  is  $x + 1$ .
  - (2)  $i \geq 1$  and the number of **a**'s among the first  $i$  characters of  $S$  is  $x$ .
  - (3)  $i \geq 1$  and the number of **a**'s among the first  $i - 1$  characters of  $S$  is  $x$ .
  - (4)  $i \geq 1$  and if  $x = 0$  then  $\ell = 0$ .
- (c) Explain (in detail) why the statement you chose in part (b) is true the first time line 5 is executed.
- (d) Let  $k \geq 1$ . Assume that when line 5 is executed for the  $k$ th time, the statement you chose in part (b) is true *and* that  $i \leq \ell$ . Then, explain (in detail) why the statement you chose in part (b) is true when line 5 is executed for the  $(k + 1)$ th time.

- (e) Assume that, at some execution of line 5, the statement you chose in part (b) is true *and* that  $i > \ell$ . Then, the loop terminates, and the algorithm returns a boolean value. Explain (in detail) why this returned value makes your answer to part (a) true.
- (f) When Egbert sees Willemina's pseudocode, he comes up with a simpler algorithm that accomplishes exactly the same task as Willemina's. He changes lines 4, 7, 11 to get the following algorithm.

```
1 MYSTERY-EGBERT(string  $S$ ) : boolean
2   int  $\ell \leftarrow \text{length}(S)$ 
3   int  $i \leftarrow 1$ 
4   boolean  $x \leftarrow \text{FALSE}$ 
5   while  $i \leq \ell$ 
6     if  $S(i) = \text{a}$  then
7        $x \leftarrow$  _____
8     end if
9      $i \leftarrow i + 1$ 
10  end while
11  return ( $x$ )
12 end MYSTERY-EGBERT
```

Provide the expression that Egbert wrote in the blank on line 7.