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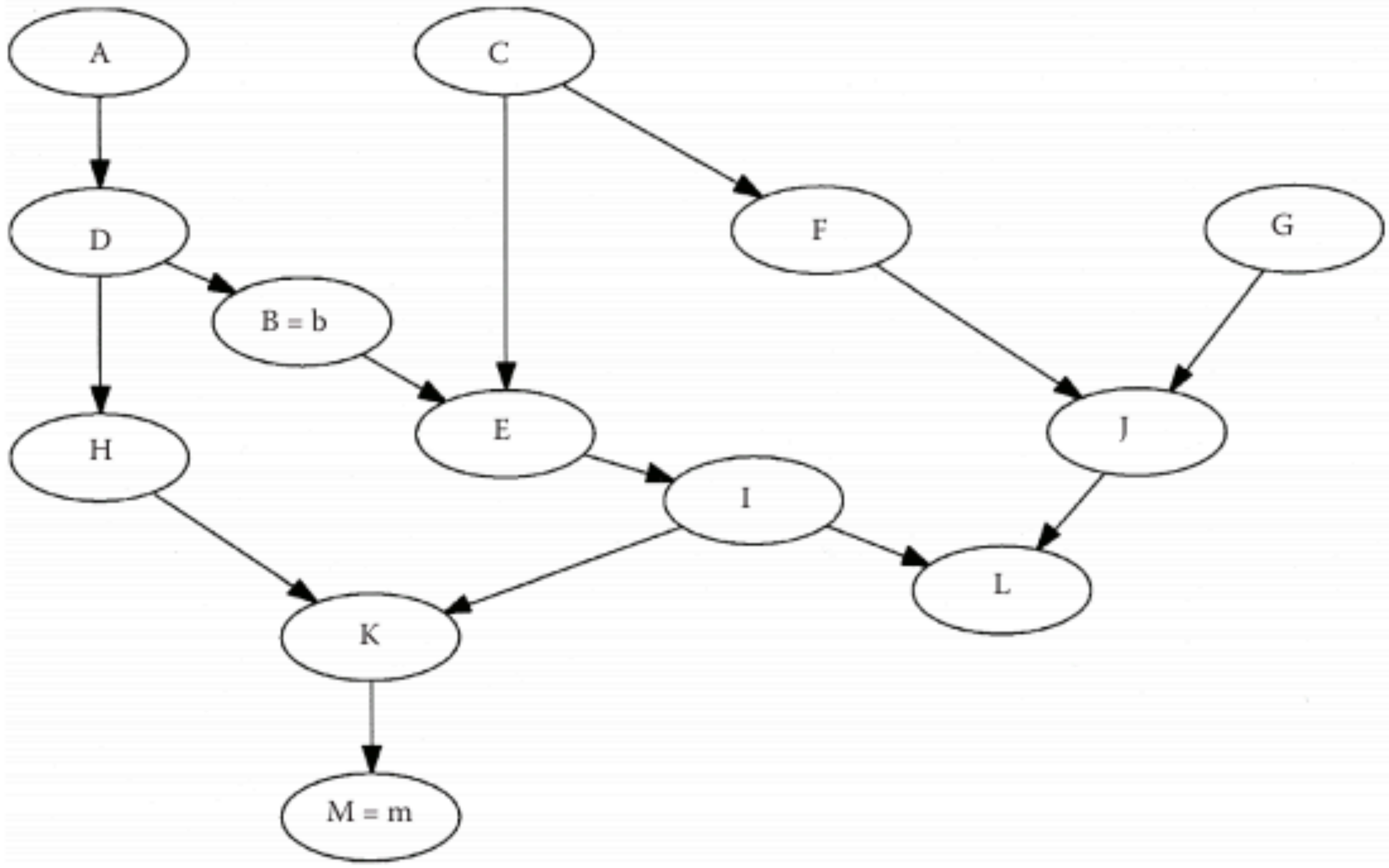

Figure 6.19 Determining whether nodes are *d*-connected.

In cases 1 and 2 we say that the nodes *B* and *C* are *d*-separated when there is hard evidence of *A*. In case 3 *B* and *C* are only *d*-separated when there is no evidence about *A*. In general two nodes that are not *d*-separated are said to be *d*-connected.

These three cases enable us to determine in general whether any two nodes in a given BN are dependent (*d*-connected) given the evidence entered in the BN. Formally:

Two nodes *X* and *Y* in a BN are *d*-separated if, for all paths between *X* and *Y*, there is an intermediate node *A* for which either:

1. The connection is serial or diverging and the state of *A* is known for certain; or
2. The connection is converging and neither *A* (nor any of its descendants) has received any evidence at all.

If *X* and *Y* are not *d*-separated then they are said to be *d*-connected.

In the BN in Figure 6.19 the evidence entered at nodes *B* and *M* represents instantiation, that is, $B = b$ and $M = m$. We start the example by entering evidence at *A* and seeing how it affects the other nodes in the BN (this is called propagation of effects). We are especially interested in whether it affects node *J* and *G* and we can trace the propagation by determining first the *d*-connections that each node belongs to and the trails through which dependencies can flow.

If evidence is entered at *A* it affects *D* and *H* and *K*. At *B* it is blocked by $B = b$.

Since *K* is part of a converging *d*-connection triad of nodes with *H* and *I* as parents any evidence at *M* influences *K* and makes *I* and *H* *d*-connected. Therefore the evidence propagated from *A* to *H* influences *I*.

I is part of a serial connection from *E* to *L*, therefore *E* and *L* change. *E* is part of a diverging *d*-connection triad with *C* and *F*, therefore changes at *E* affect *C* and *F* and through to *J*.

Does node *G* change? A descendent of node *J* has been updated, node *L*, but crucially has not received either hard or uncertain evidence directly itself or on a direct descendant, therefore *G* and *F* are *d*-separated and *G* remains unchanged. Therefore *G* is independent of *A* given $B = b$, $M = m$.