

Quiz

First Name: _____

Last Name: _____

Student Number: _____

This test lasts 45 minutes. No aids allowed.

Make sure your test has 3 pages, including this cover page.

*Answer in the space provided. (If you need more space, use the reverse side of the page and indicate **clearly** which part of your work should be marked.)*

Write legibly.

Question 1	/4
Question 2	/4
Question 3	/4
Question 4	/3
Total	/15

[4] **1.** If $A = \{1, 2, 3\}$ and $B = \{a, b\}$, list all elements of the following sets:

(a) $A \times B =$

(b) The power set of B , $\mathcal{P}(B) =$

[4] **2.** For every $i \in \mathbb{N}$, we define a string s_i as follows.

$$s_0 = \text{ba}$$

$$s_1 = \text{ab}$$

$$s_i = s_{i-1} \cdot s_{i-2}, \text{ for } i \geq 2$$

For example, $s_3 = \text{abbaab}$, which contains three a's and three b's. Give a careful proof that, for all $i \geq 0$, the string s_i contains equal numbers of a's and b's.

- [4] **3.** Let $L = \{x \in \{a, b\}^* : x \text{ contains at most three a's}\}$. Draw the transition diagram of a DFA for L . For each state of your machine, give a precise description of the strings that take the machine to that state.

- [3] **4.** Is it true that $L_1^* \cap L_2^* = (L_1 \cap L_2)^*$ for all languages L_1 and L_2 ? Prove your answer is correct.