Test 1

First Name: ____________________

Last Name: ____________________

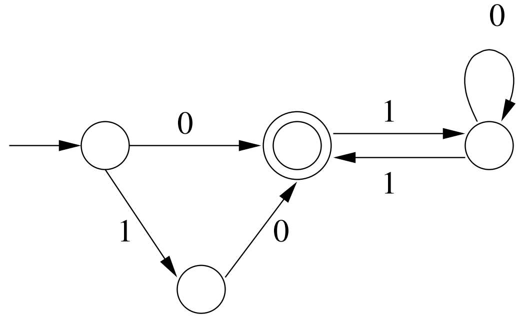
Student Number: ____________________

This test lasts 80 minutes. *No aids allowed.*
You may use any result that was proved in class or in the textbook without reproving it.
Make sure your test has 5 pages, including this cover page.
Answer in the space provided. (If you need more space, use the reverse side of the page and indicate ***clearly*** *which part of your work should be marked.)*
Write legibly.

Question 1	/4
Question 2	/3
Question 3	/3
Question 4	/4
Question 5	/3
Question 6	/3
Total	/20

[4] **1.** Let $L = \{w \in \{a, b\}^* : w$ does not contain the substring $aab\}$. For example, $babb$ and ba are in L, but $bbaaba$ is not in L. Draw a deterministic finite automaton for L.

[3] **2.** Write a regular expression that describes the language accepted by the automaton shown below.

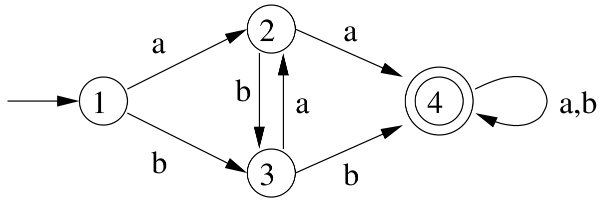


[3] **3.** Let $\Sigma = \{a, b, c\}$. We define a function $f : \Sigma^* \to \Sigma^*$ as follows. For any $x \in \Sigma^*$, $f(x)$ is the string obtained by replacing each occurrence of b in x by cc. For example, $f(ab) = acc$, $f(cab) = cacc$ and $f(bc) = ccc$.

Suppose $M = (Q, \Sigma, \delta, q_0, F)$ is a deterministic finite automaton for some language L. Give a formal description of a finite automaton for the language $\{f(x) : x \in L\}$.

(You do *not* have to prove your answer is correct.)

[4] **4.** Consider the deterministic finite automaton M shown below.



Lemma 1: Every string that takes M to state 2 ends in a.
Lemma 2: Every string that takes M to state 3 ends in b.
Lemma 3: Every string that M accepts contains two consecutive characters that are the same.
Assume that Lemma 1 and Lemma 2 have already been proved.
Give a careful proof of Lemma 3. Hint: You can use induction on the length of the string.

[3] **5.** Let L be the language of all binary strings of odd length whose middle character is 0. (More formally, $L = \{x0y : x, y \in \{0,1\}^* \text{ and } |x| = |y|\}$.) Is L regular? Prove your answer is correct.

[3] **6.** If L is a language over the alphabet Σ, the extension of L is

$$EXT(L) = \{x \in \Sigma^* : \text{ some prefix of } x \text{ is in } L\}.$$

Prove the following claim: for every regular language L, $EXT(L)$ is also regular.