Homework Assignment #8
Due: Thursday, November 13, 2014 at 4:00 p.m.

You may use the Church-Turing thesis freely on this assignment.

1. Is the following statement true? If L is a decidable language and $L' \subseteq L$, then L' is also decidable. Prove your answer is correct.

2. Let $SUB_{TM} = \{\langle M_1, M_2\rangle : M_1 \text{ and } M_2 \text{ are Turing machines and } L(M_1) \subseteq L(M_2)\}$.

(a) Suppose you had an algorithm $S(\langle M_1, M_2\rangle)$ that decides the language SUB_{TM}. Show how you could use this algorithm as a subroutine to design an algorithm $A(\langle M_1, w\rangle)$ that decides the language A_{TM}.

(b) Use Theorem 4.11 of the textbook to prove that SUB_{TM} is undecidable.

Bonus Question: Busy Beavers
The bonus question below is separate from the rest of Assignment 8.
The bonus question is due Thursday, November 20 at 11:59 p.m.

3. In this question, we shall consider the basic Turing machines as defined in Assignment 6. We shall restrict attention to machines that use the tape alphabet $\Gamma = \{0, 1, 2, \sqcup, \triangleright\}$. The goal is to design a Turing machine which, on input ε, takes as many steps as possible before halting. Your machine must have exactly six states, including the initial state, the accepting state and the rejecting state.

This bonus question is worth up to 4 percent to be added to your final grade. Students who submit a correct Turing machine will get between 1 and 4 bonus points. If two students submit machines that take the same number of steps, each student will get 1 point. Excluding these students, the higher the number of steps your machine takes, the more bonus points you will earn. To earn all 4 points, you will have to make your machine take a very large number of steps.

If your machine takes more than 1,000,000,000 steps on the empty input string, I may not be able to test it by running it, so you will have to provide, in writing, a proof that it does indeed halt (and that it takes lots of steps) in order to receive your bonus points.

Write your Turing machine in a YUTMFF text file named a8.txt and submit it online at `https://webapp.eecs.yorku.ca/submit/` or by using the following command.

```
submit 2001 a8 a8.txt
```

If your submission is syntactically incorrect or does not follow the specifications of this question (for example if it has too many states) or runs forever on the empty input string, you will receive 0 points.

For this bonus question, you may consult outside sources. A good place to start is the Computer Recreations column by A. K. Dewdney in the August, 1984 issue of *Scientific American* (pp. 19–23). (Note that the Turing machines discussed there use two-way infinite tapes, so you cannot use them directly to answer this question, but you might get some ideas about how to design your own busy beaver.)