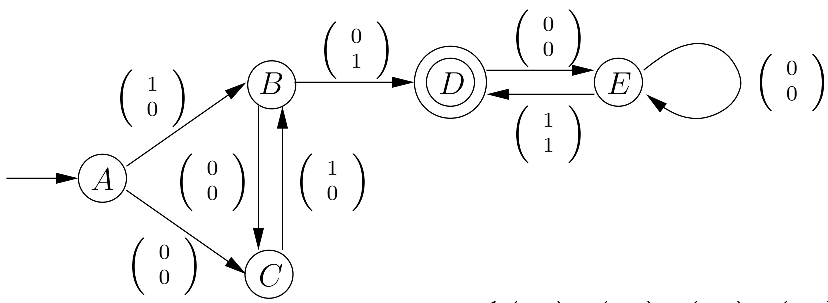
Homework Assignment #3
Due: October 2, 2014 at 4:00 p.m.

1. Draw the transition diagram of a deterministic finite automaton that accepts the language of all binary strings that contain an even number of 0's and end with a 1.

You do not have to prove your answer is correct, but you should state, for each state of your automaton, a precise description in English of exactly which strings can take the automaton to that state.

2. Consider the deterministic finite automaton $(Q, \Sigma, \delta, q_0, F)$ given in the following diagram.



Note that the input alphabet Σ is $\left\{\binom{0}{0}, \binom{0}{1}, \binom{1}{0}, \binom{1}{1}\right\}$.

We extend the definition of δ to strings recursively as follows:

$$\begin{aligned}\delta^*(q, \varepsilon) &= q \text{ for all } q \in Q \\ \delta^*(q, xa) &= \delta(\delta^*(q, x), a) \text{ for all } q \in Q, x \in \Sigma^*, a \in \Sigma\end{aligned}$$

Recall the Leutonian representation of numbers used in Assignment 2. Given a string $s \in \Sigma^*$, we define *top*(s) and *bottom*(s) to be the numbers represented in Leutonian notation by the *reverse* of the top and bottom row of bits in s. For example, if

$$s = \binom{1}{0}\binom{0}{0}\binom{0}{1}\binom{1}{0}\binom{0}{0}\binom{1}{1},$$

then *top*$(s) = n(101001) = 19$ and *bottom*$(s) = n(100100) = 16$ where n is the function defined in Assignment 2. Here, we allow leading 0's in the Leutonian representation of a number, which do not affect the value represented. For example, $n(00101) = 4$.

(a) If s is a string of length ℓ and $\delta^*(A, s) = B$, state a very simple arithmetic expression (in terms of ℓ) for *top*(s).

(b) If s is a string of length ℓ and $\delta^*(A, s) = C$, state a very simple arithmetic expression (in terms of ℓ) for *top*(s).

(c) Claim: For all $\ell \geq 1$, and every string s of length ℓ, my answers to part (a) and (b) are correct.

Prove this claim by induction on n.

(d) Give a simple relationship between *top*(s) and *bottom*(s) that is true if and only if s accepted by the automaton. You do not have to prove your answer is correct.