Homework Assignment #1
Due: September 18, 2014 at 4:00 p.m.

This assignment is based on material that is a prerequisite to this course. See the handout "Mathematical Prerequisites". After doing any rough work required, you should write your solutions on this document in the spaces provided and then hand it in. (Most of the assignments later in this course will have fewer, but longer, questions; this assignment is just intended to give you quick feedback on whether you need to review some of the background material more thoroughly.)

Name: ____

Student Number: ___________________________

1. The last two digits of your student number form a decimal number between 0 and 99. Write down the binary representation of that number.

2. Let the predicate $E(x, y)$ represent the statement "Person x eats food y".
Let the predicate $M(y)$ represent the statement "Food y is a meat product".

(a) Express the following statement in predicate logic: "Someone is a vegetarian."

(b) Express the following statement in predicate logic: "Nobody (except maybe John) eats lasagna."

(c) Explain the difference between the statements "$\forall y \exists x E(x, y)$" and "$\exists x \forall y E(x, y)$".

3. For each of the following sets, determine whether the set is finite or infinite. If the set is finite, write down an explicit list of all the elements in the set. If the set is infinite, say so and list five elements of the set. ($\mathbb{N}$ denotes the set of non-negative integers.)

(a) $A = \{n \in \mathbb{N} : \forall m \leq n, n \neq m^2\}$

(b) $B = \{n \in \mathbb{N} : \forall m \leq n, n = m^2\}$

(c) $C = \{n^2 : n \in \mathbb{N} \text{ and } 2 < n < 6\}$.

(d) $D = \{(n, m) \in \mathbb{N} \times \mathbb{N} : 1 < n < m < 6\}$.

(e) $E = \mathscr{P}(\{b, c, d\})$

4. If $w : \{1..\ell\} \to \{0, 1\}$ is a binary string, the complement of w, denoted w^C, is the string of length ℓ defined by $w^C(i) = 1 - w(i)$. The reverse of w, denoted w^R, is the string of length ℓ defined by $w^R(i) = w(\ell + 1 - i)$. Use these definitions to give a careful proof that, for every binary string x, $(x^C)^R = (x^R)^C$.

5. A programming exercise.

Let $\Sigma = \{), (\}$. Let $L \subseteq \Sigma^*$ be the set of strings of correctly balanced parentheses. For example, (())() is in L and (()))(is not in L. Formally, L is defined recursively as follows.

- $\varepsilon \in L$.
- A string $x \neq \varepsilon$ is in L if and only if x is of the form $(y)z$, where y and z are in L.

Let $f(n)$ be the number of strings in L that have length at most n.

The last 3 digits of your student number form a 3-digit number n between 0 and 999. Compute $f(n)$ mod 997. You do not have to prove your answer is correct.

Some facts you might find useful: if $n_1, n_2 \in \mathbb{N}$, then

- $(n_1 n_2) \bmod 997 = ((n_1 \bmod 997)(n_2 \bmod 997)) \bmod 997$ and
- $(n_1 + n_2) \bmod 997 = ((n_1 \bmod 997) + (n_2 \bmod 997)) \bmod 997$.

Answer: $f($_______$)$ mod 997 = _______