YORK UNIVERSITY

FACULTY OF SCIENCE AND ENGINEERING

2012 SUMMER TERM EXAMINATION

Course: CSE2001 Introduction to the Theory of Computation

Duration: 3 hours

No aids allowed.

- There should be 11 pages in the exam, including this page.
- Write all answers on the examination paper. If your answer does not fit in the space provided, you can continue your answer on the back of a page or on page 11, indicating clearly that you have done so.
- You may use any result that was proved in class or the readings without reproving it.
- Please write legibly.

Name ______________________________

Student Number ______________________________

Question 1	/8
Question 2	/4
Question 3	/3
Question 4	/4
Question 5	/3
Question 6	/3
Question 7	/4

Question 8	/2
Question 9	/2
Question 10	/3
Question 11	/5
Question 12	/4
Question 13	/5
Bonus	/4
Total	/50

[8] **1.** For each of the following languages, you must determine whether the language is regular, context-free, decidable, recognizable or not recognizable. For each language, circle the *leftmost* correct answer. For example, if a language is both recognizable and decidable, but not context-free, circle decidable.

(a) $\{1^p : p \text{ is a prime number}\}$

regular context-free decidable recognizable not recognizable

(b) $\{0^n 1^n : n \geq 0\}$

regular context-free decidable recognizable not recognizable

(c) $\{x \in \{0,1\}^* : x \text{ contains an even number of 0's}\}$

regular context-free decidable recognizable not recognizable

(d) $\{x \in \{0,1\}^* : x \text{ contains equal numbers of 0's and 1's}\}$

regular context-free decidable recognizable not recognizable

(e) $\{\langle M \rangle : M \text{ is a Turing machine that accepts at least three different input strings}\}$

regular context-free decidable recognizable not recognizable

(f) $\{\langle M \rangle : M \text{ is a Turing machine and for all } n \geq 0,\ M \text{ accepts some string of length } n\}$

regular context-free decidable recognizable not recognizable

(g) $\{\langle G, w \rangle : G \text{ is a context-free grammar that generates string } w\}$

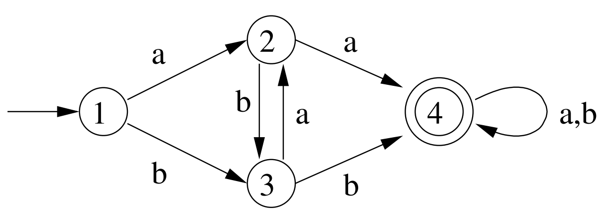
regular context-free decidable recognizable not recognizable

(h) $\{\langle M \rangle : M \text{ is the only Turing machine that accepts } L(M)\}$

regular context-free decidable recognizable not recognizable

[4] **2.** Let $L_2 = \{w \in \{a,b\}^* : \text{each letter in } w \text{ appears at least twice in } w\}$. For example, *ababb* and *bb* are in L_2, but *abbb* is not in L_2. Draw a deterministic finite automaton for L_2.

[3] **3.** Give a regular expression for the language accepted by the finite automaton shown below. You do not have to prove your answer is correct.



[4] **4.** Let $L_4 = \{0^i 1^j : i > j > 0\}$. Is L_4 regular? Prove your answer is correct.

[3] **5.** Give an example of a non-regular language L such that L^* is regular.
Explain why your answer is correct.

[3] **6.** Let $L_6 = \{0^i 1^j 2^k : i = j \text{ or } j = k\}$.
Briefly describe, in English, how a pushdown automaton that accepts L_6 would work.
Your answer must fit in the space below.

[4] **7.** Let $L_7 = \{w\#x : w, x \in \{0,1\}^* \text{ and } w^R \text{ is a substring of } x\}$.
For example, 011#1111010 is in L_7 because 110 is a substring of 1111010.
Give a context-free grammar for L_7.
You do not have to prove your answer is correct. However, you should give a precise description of the strings that are generated by each of your grammar's variables.

[2] **8.** Let $L = \{a^i b^{i+j} c^j : i, j \geq 0\}$ and let G be a context-free grammar.
You could prove that L is the language generated by G by proving *two* of the following statements by induction on n. Which two? (Circle the numbers of the two statements.)

1. For all $n \geq 0$, some string of length n in L is generated by G.
2. For some $n \geq 0$, every string of length n in L is generated by G.
3. For all $n \geq 0$, G generates all strings of length n that are in L.
4. For all $n \geq 0$ and all strings x, if G generates x in n steps, then $x \in L$.
5. For all $n \geq 0$ and all strings $x \in L$, G generates x in n steps.

[2] **9.** Give a precise definition of $L \leq_m L'$. (I.e., what does it mean for a language L to be many-one reducible to L'?)

[3] **10.** Suppose, after graduating from York, you go to work for a company that is creating a new Java compiler. To make the compiled code run faster, the compiler automatically performs some optimizations. For example, if a programme contains the line

```
x = f(y) * f(y);
```

the compiler will change it to

```
temp = f(y);
x = temp * temp;
```

to save the cost of evaluating `f(x)` twice. (Here, `temp` is a variable that does not appear anywhere else in the programme being optimized and it will be suitably declared.)

Your job is in the testing department. To test the optimizations performed by the compiler, you must check that the optimized version of the code behaves the same way as the original code. Your boss tells you to write a testing programme that takes, as input, A and B, where A is a chunk of Java code and B is the optimized version produced by the compiler's optimization algorithm. Your programme should output YES if A and B produce exactly the same results under all circumstances, and NO otherwise. Briefly describe how you would respond to your boss's instructions.

[5] **11.** Suppose L and L' are two languages over the same alphabet Σ.
Recall that $L - L' = \{w \in L : w \notin L'\}$.

[2] **(a)** Show that if L and L' are decidable, then $L - L'$ is also decidable.

[3] **(b)** Give examples of two recognizable languages L and L' such that $L - L'$ is not recognizable.
Explain why your answer is correct.

[4] **12.** Let $L_{12} = \{\langle M \rangle : M \text{ is a Turing machine that runs forever on input } \varepsilon\}$.
Prove that L_{12} is not recognizable.

[5] **13.** Let $L_{13} = \{\langle M \rangle$: for some input string, Turing machine M reaches its seventh state$\}$. (If M has fewer than 7 states, then $\langle M \rangle \notin L_{13}$.)

[1] **(a)** Briefly state why Rice's Theorem does not apply to L_{13}.

[4] **(b)** Prove that L_{13} is recognizable but not decidable.

[4] **14. BONUS question (attempt only if you have extra time)**:
Let $L_{14} = \{\langle M \rangle : \text{on input } \varepsilon, M \text{ eventually visits the 100th square on its tape}\}$.
(Here we use the basic definition of a Turing machine with a one-way infinite tape.)
Show that L_{14} is decidable.
Hint: how many different configurations are there for a machine that never visits the 100th square?

This (nearly) blank page is for extra work space.