

SC/MATH 1090

10- The Existential Quantifier

Ref: G. Tourlakis, *Mathematical Logic*, John Wiley & Sons, 2008.

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Overview

- Insert/Remove the existential quantifier
 - Dual of spec
 - \forall Introduction vs \exists Introduction
 - Auxiliary Variable Metatheorem

Some Duals

- **Theorem. (Dual of Ax2)** $\vdash A[x:=t] \rightarrow (\exists x)A$
- **Corollary. (Dual of Specialization Rule)** $A[x:=t] \vdash (\exists x)A$
- **Corollary.** $A \vdash (\exists x)A$

- **Metatheorem. (\forall Introduction)** If x does not occur either in Γ or in A , then $\Gamma \vdash A \rightarrow B$ iff $\Gamma \vdash A \rightarrow (\forall x)B$.
- **Metatheorem. (\exists Introduction)** If x does not occur either in Γ or in B , then $\Gamma \vdash A \rightarrow B$ iff $\Gamma \vdash (\exists x)A \rightarrow B$.

Auxiliary Variable Metatheorem

- Assume that $\Gamma \vdash (\exists x)A$.
- Moreover assume that $\Gamma + A[x:=z] \vdash B$,
(z fresh with respect to $\Gamma, (\exists x)A, B$)
- Then $\Gamma \vdash B$.
- **Important:** Note $A[x:=z]$, i.e. the **auxiliary hypothesis**, is an extra hypothesis to check for free variables, in case you are using **weak generalization** later in your proof.

Corollaries of Auxiliary Variable Metatheorem

- **Corollary.** Assume that $\vdash (\exists x)A$. Moreover assume that $A[x:=z] \vdash B$, where z fresh with respect to $(\exists x)A$ and B . Then $\vdash B$.
- **Corollary.** Assume $A[x:=z] \vdash B$, where z fresh with respect to $(\exists x)A$ and B . Then $(\exists x)A \vdash B$.