Axioms of Boolean Logic

Associativity of $\equiv$	$((A \equiv B) \equiv C) \equiv (A \equiv (B \equiv C))$	(1)
Symmetry of $\equiv$	$(A \equiv B) \equiv (B \equiv A)$	(2)
$\top$vs. $\bot$	$\top \equiv \bot \equiv \bot$	(3)
introduction of $\neg$	$\neg A \equiv A \equiv \bot$	(4)
Associativity of $\vee$	$(A \vee B) \vee C \equiv A \vee (B \vee C)$	(5)
Symmetry of $\vee$	$A \vee B \equiv B \vee A$	(6)
Idempotency of $\vee$	$A \vee A \equiv A$	(7)
Distributivity of $\vee$ over $\equiv$	$A \vee (B \equiv C) \equiv A \vee B \equiv A \vee C$	(8)
Excluded Middle	$A \vee \neg A$	(9)
Golden Rule	$A \wedge B \equiv A \equiv B \equiv A \vee B$	(10)
Implication	$A \to B \equiv A \vee B \equiv B$	(11)

Some Theorems of Boolean Logic

Transitivity	$A \equiv B, B \equiv C \vdash A \equiv C$	$(1.4.13(c))$
	$\vdash A \equiv A$	$(1.4.13(d))$
	$\vdash \top$	$(2.1.15)$
Eqn + Leib Merged	$C[\mathbf{p} := A], A \equiv B \vdash C[\mathbf{p} := B]$	$(2.1.16)$
Redundant True	$\vdash \top \equiv A \equiv A$	$(2.1.21)$
	$\vdash \neg(A \equiv B) \equiv \neg A \equiv B$	$(2.4.1)$
Double Negation	$\vdash \neg\neg A \equiv A$	$(2.4.4)$
	$\vdash \top \equiv \neg\bot$	$(2.4.5)$
	$\vdash A \vee \top$	$(2.4.7)$
	$\vdash A \vee \bot \equiv A$	$(2.4.10)$
	$\vdash A \to B \equiv \neg A \vee B$	$(2.4.11)$
	$\vdash \neg A \vee B \equiv A \vee B \equiv B$	$(2.4.12)$
	$\vdash A \to (B \equiv C) \equiv A \to B \equiv A \to C$	$(2.4.13)$
de Morgan 1	$\vdash A \wedge B \equiv \neg(\neg A \vee \neg B)$	$(2.4.17)$
de Morgan 2	$\vdash A \vee B \equiv \neg(\neg A \wedge \neg B)$	$(2.4.18)$
	$\vdash A \wedge A \equiv A$	$(2.4.19)$
	$\vdash A \wedge \top \equiv A$	$(2.4.20)$
	$\vdash A \wedge \bot \equiv \bot$	$(2.4.21)$

Some Theorems of Boolean Logic- continued

Distributivity of $\vee$ over $\wedge$	$\vdash A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$	$(2.4.23(i))$
Distributivity of $\wedge$ over $\vee$	$\vdash A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$	$(2.4.23(ii))$
Distributivity of $\rightarrow$ over $\wedge$	$\vdash A \rightarrow (B \wedge C) \equiv (A \rightarrow B) \wedge (A \rightarrow C)$	$(2.4.25)$
	$\vdash (A \vee B) \rightarrow C \equiv (A \rightarrow C) \wedge (B \rightarrow C)$	$(2.4.24)$
Ping-Pong Theorem	$\vdash A \equiv B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$	$(2.4.26)$
Merge	$A, B \vdash A \wedge B$	$(2.5.1(1))$
	$A \vee A \vdash A$	$(2.5.1(2))$
	$A \vdash A \vee B$	$(2.5.1(3))$
Split	$A \wedge B \vdash A$	$(2.5.1(4))$
Cut Rule	$A \vee B, \neg A \vee C \vdash B \vee C$	$(2.5.4)$
	$A \vee B, \neg A \vee B \vdash B$	$(2.5.5)$
	$A \vee B, \neg A \vdash B$	$(2.5.6)$
	$A, \neg A \vdash \bot$	$(2.5.7)$
Modus Ponens	$A, A \rightarrow B \vdash B$	$(2.5.3)$
Transitivity of $\rightarrow$	$A \rightarrow B, B \rightarrow C \vdash A \rightarrow C$	$(2.5.9)$
	$A \rightarrow C, B \rightarrow D \vdash A \vee B \rightarrow C \vee D$	$(2.5.10)$
Proof by Cases	$A \rightarrow C, B \rightarrow C \vdash A \vee B \rightarrow C$	$(2.5.11)$
	$A \rightarrow C, \neg A \rightarrow C \vdash C$	$(2.5.13)$

Axioms of Predicate Logic

The axioms of Predicate Logic consist of all partial generalizations of the following schemata:

Ax1	All tautologies.	
Ax2	$(\forall x)A \to A[x := t]$	Specialization axiom
Ax3	$(\forall x)(A \to B) \to (\forall x)A \to (\forall x)B$	
Ax4	$A \to (\forall x)A$, if x dnof in A	
Ax5	$x = x$	Identity axiom
Ax6	$s = t \to (A[x := s] \equiv A[x := t])$	Leibniz axiom for equality

Some theorems $\&$ metatheorems of Predicate Logic

Weak Generalization	If $\Gamma \vdash A$, then $\Gamma \vdash (\forall x)A$ if x dnof in any formula of Γ	(6.1.1)
	If $\vdash A$, then $\vdash (\forall x)A$	(6.1.3)
Specialization	$(\forall x)A \vdash A[x := t]$	(6.1.5)
	$(\forall x)A \vdash A$	(6.1.6)
	$\vdash (\forall x)(\forall y)A \equiv (\forall y)(\forall x)A$	(6.1.8)
Distributivity of $\forall$ over $\wedge$	$\vdash (\forall x)(A \wedge B) \equiv (\forall x)A \wedge (\forall x)B$	(6.1.7)
$\forall$-monotonicity	If $\Gamma \vdash A \to B$, then $\Gamma \vdash (\forall x)A \to (\forall x)B$ if x dnof in any formula of Γ	(6.1.9)
	If $\Gamma \vdash A \equiv B$, then $\Gamma \vdash (\forall x)A \equiv (\forall x)B$ if x dnof in any formula of Γ	(6.1.11)
Weak Leibniz	If $\vdash A \equiv B$, then $\vdash C[\mathbf{p} \backslash A] \equiv C[\mathbf{p} \backslash B]$	(6.2.1)
Strong Leibniz	$A \equiv B \vdash C[\mathbf{p} := A] \equiv C[\mathbf{p} := B]$	(6.2.3)
	$\vdash (\forall x)(A \to B) \equiv A \to (\forall x)B$ if x dnof in$A$	(6.4.1)
	$\vdash (\forall x)(A \vee B) \equiv A \vee (\forall x)B$ if x dnof in$A$	(6.4.2)
	$\vdash (\exists x)(A \wedge B) \equiv A \wedge (\exists x)B$ if x dnof in$A$	(6.4.3)

Some theorems & metatheorems of Predicate Logic- continued

Dummy renaming	$\vdash (\forall x)A \equiv (\forall z)A[x := z]$ if z is fresh to A	(6.4.4)
	$\vdash (\exists x)A \equiv (\exists z)A[x := z]$ if z is fresh to A	(6.4.5)
Dual of Ax2 (Spec axiom)	$\vdash A[x := t] \to (\exists x)A$	(6.5.1)
Dual of Spec rule	$A[x := t] \vdash (\exists x)A$	(6.5.2)
	$A \vdash (\exists x)A$	(6.5.3)
$\forall$ Introduction	$\Gamma \vdash A \to B$ iff $\Gamma \vdash A \to (\forall x)B$ if x dnof in A or Γ	(6.5.4)
$\exists$ Introduction	$\Gamma \vdash A \to B$ iff $\Gamma \vdash (\exists x)A \to B$ if x dnof in B or Γ	(6.5.5)
Auxiliary Variable	If $\Gamma \vdash (\exists x)A$ and $\Gamma + A[x := z] \vdash B$ then $\Gamma \vdash B$if z is fresh	(6.5.6)
Soundness in first order logic	If $\vdash A$ then $\models A$	(8.2.3)
Completeness in first order logic	If $\models A$ then $\vdash A$	(8.2.4)