MATH/EECS 1019 First test (version 2)
Fall 2014
Solutions
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1. (6 points) Propositional Logic.

   (a) (2 points) Construct a truth table for the implication $\neg p \leftrightarrow \neg q$

   Solution:

p	q	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

   (b) (2 points) Let p be the proposition "You have the flu", q be the proposition "You miss the final examination" and r be the proposition "You pass the course". Express the following as an English sentence: $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$.

   Solution: "If you have the flu then you do not pass the course or if you miss the final examination then you fo not pass the course."

   Or more simply, "If you have the flu or if you miss the final examination then you do not pass the course".

   (c) (2 points) Let p be the proposition "You get an A on the final exam", q be the proposition "You do every exercise in the book" and r be the proposition "You get an A in this course". Write down the following using p, q and r and logical connectives (including negations): "You get an A on the final, but you don't do every exercise in the book; nevertheless you get an A in this class".

   Solution: This translates to $p \wedge \neg q \wedge r$.

2. (6 points) Propositional equivalences

   (a) (3 points) Use truth tables to verify the absorption law: $p \wedge (p \vee q) \equiv p$

   Solution:

p	q	$(p \vee q)$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

   Since the columns for p and $p \wedge (p \vee q)$ are identical in the truth table, they must be logically equivalent.

   (b) (3 points) Show that $\neg(p \leftrightarrow q)$ and $(p \leftrightarrow \neg q)$ are logically equivalent.

   Solution: This can be done with truth tables or analytically. I will do the analytical proof here.

$$\begin{aligned}
\neg(p \leftrightarrow q) &\equiv \neg((p \rightarrow q) \wedge (q \rightarrow p)) \\
&\equiv \neg((\neg p \vee q) \wedge (\neg q \vee p)) \\
&\equiv \neg(\neg p \vee q) \vee \neg(\neg q \vee p), \text{ De Morgan's law} \\
&\equiv (p \wedge \neg q) \vee (q \wedge \neg p), \text{ De Morgan's law}
\end{aligned}$$

$$
\begin{aligned}
p \leftrightarrow \neg q &\equiv (p \rightarrow \neg q) \wedge (\neg q \rightarrow p) \\
&\equiv (\neg p \vee \neg q) \wedge (q \vee p) \\
&\equiv ((\neg p \vee \neg q) \wedge q) \vee ((\neg p \vee \neg q) \wedge p), \text{ Distributive law} \\
&\equiv ((\neg p \wedge q) \vee (\neg q \wedge q)) \vee ((\neg p \wedge p) \vee (\neg q \wedge p)), \text{ Distributive law} \\
&\equiv ((\neg p \wedge q) \vee F) \vee (F \vee (\neg q \wedge p)), \text{ Negation law (twice)} \\
&\equiv (\neg p \wedge q) \vee (\neg q \wedge p), \text{ Domination law (twice)}
\end{aligned}
$$

Since both expressions are equivalent to the same same expression, they are logically equivalent.

3. (6 points) Predicates.

   (a) (2 points) Determine the truth value of the following statement, giving reasons. The domain is all real numbers. $\forall x(2x > x)$
   Solution: This is False. A counterexample is $x = -1$ for which $2x = -2$ and so $2x < x$.

   (b) (2 points) Express using logical operators, quantifiers and predicates: "The conjunction of two tautologies is a tautology".
   Solution from the text: Let $T(x)$ mean that x is a tautulogy. Then $\forall x \forall y((T(x) \wedge T(y)) \rightarrow T(x \wedge y)$.
   Note: The solution should also mention that the domain is all propositions. Since x, y are propositions, $x \wedge y$ is well defined.

   (c) (2 points) Let $P(x), Q(x), R(x), S(x)$ be the statements "x is a baby", "x is logical", "x is able to manage a crocodile' and "x is despised"' respectively. Suppose that the domain consists of all people. Express the following using quantifiers and the above predicates: "Illogical persons are despised".
   Solution from the text: $\forall x(\neg Q(x) \rightarrow S(x))$.

4. (6 points) Nested quantifiers

   (a) (2 points) Let $F(x, y)$ be the statement "x can fool y", where the domain consists of all people in the world. Use quantifiers to express the following statement: "There is no one who can fool everybody".
   Solution: Translated literally the sentence is $\neg(\exists x \forall y \exists F(x, y))$. We can simplify this to $\forall x \exists y \neg F(x, y)$.

   (b) (2 points) Express the following statement in predicate logic: "the product of two negative real numbers is positive".
   Solution from the text: $\forall x \forall y((x < 0) \wedge (y < 0)) \rightarrow (xy > 0))$.

   (c) (2 points) Express the negative of the following statement so that all negation symbols immediately precede predicates.
   $\exists x \forall y(P(x, y) \rightarrow Q(x, y))$
   Solution:

$$
\begin{aligned}
\neg(\exists x \forall y(P(x, y) \rightarrow Q(x, y))) &\equiv \forall x \neg(\forall y(P(x, y) \rightarrow Q(x, y))) \\
&\equiv \forall x \exists y(\neg(P(x, y) \rightarrow Q(x, y))) \\
&\equiv \forall x \exists y(\neg(\neg P(x, y) \vee Q(x, y))) \\
&\equiv \forall x \exists y(P(x, y) \wedge \neg Q(x, y))
\end{aligned}
$$

5. (6 points) Inference.

(a) (3 points) For the following premises, what relevant conclusion(s) can be drawn? Explain the rules of inference used to obtain each conclusion.

1. What is good for corporations is good for the United States.
2. What is good for the United States is good for you.
3. What is good for the corporations is for you to buy lots of stuff.

Solution: You can keep the sentences in English or convert them to propositional logic.

1.	What is good for the corporations is for you to buy lots of stuff	Premise
2.	What is good for corporations is good for the United States	Premise
3.	It is good for the United States if you buy lots of stuff	Hypothetical Syllogism from (2) and (1)
4.	What is good for the United States is good for you	Premise
5.	It is good for you if you buy lots of stuff	Hypothetical Syllogism from (3) and (4)

(b) (3 points) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x(\neg P(x) \wedge Q(x) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is true where the domains of all quantifiers is the same.

Solution: Let c be an arbitrary element in the domain.

1.	$\forall x(P(x) \vee Q(x))$	Premise
2.	$P(c) \vee Q(c)$	Universal instantiation from (1)
3.	$\forall x(\neg P(x) \wedge Q(x) \rightarrow R(x))$	Premise
4.	$\neg P(c) \wedge Q(c) \rightarrow R(c)$	Universal instantiation from (3)
5.	$\neg(\neg P(c) \wedge Q(c)) \vee R(c)$	Equivalent form of implication
6.	$P(c) \vee \neg Q(c) \vee R(c)$	De Morgan's Law
7.	$\neg P(c) \vee R(c)$	Resolution from (2) and (6)
8.	$\neg(\neg R(c)) \vee P(c)$	Rewriting (7)
9.	$\neg R(c) \rightarrow P(c)$	Equivalent form of implication
10.	$\forall x(\neg R(x) \rightarrow P(x))$	Universal generalization from (9)