MATH/EECS 1019 First test (version 1)
Fall 2014
Solutions
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1. (6 points) Propositional Logic.

 (a) (2 points) Construct a truth table for the implication $p \to \neg q$

 Solution:

p	q	$\neg q$	$p \to \neg q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

 (b) (2 points) Let p be the proposition "You have the flu", q be the proposition "You miss the final examination" and r be the proposition "You pass the course". Express the following as an English sentence: $(p \to \neg r) \vee (q \to \neg r)$.

 Solution: "If you have the flu then you do not pass the course or if you miss the final examination then you fo not pass the course."

 Or more simply, "If you have the flu or if you miss the final examination then you do not pass the course".

 (c) (2 points) Let p be the proposition "You get an A on the final exam", q be the proposition "You do every exercise in the book" and r be the proposition "You get an A in this course". Write down the following using p, q and r and logical connectives (including negations): "Getting an A on the final and doing every exercise in the book is sufficient for getting an A in this class".

 Solution: This translates to $p \wedge q \to r$. Of course there are many equivalent expressions, e.g. $\neg(p \wedge q) \vee r$ or $\neg p \vee \neg q \vee r$.

2. (6 points) Propositional equivalences

 (a) (3 points) Use truth tables to verify the absorption law: $p \vee (p \wedge q) \equiv p$

 Solution:

p	q	$(p \wedge q)$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

 Since the columns for p and $p \vee (p \wedge q)$ are identical in the truth table, they must be logically equivalent.

 (b) (3 points) Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

 Solution: This can be done with truth tables or analytically. I will do the analytical proof here.

$$\begin{aligned}
p \leftrightarrow q &\equiv (p \to q) \wedge (q \to p) \\
&\equiv (\neg p \vee q) \wedge (\neg q \vee p) \\
&\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p), \text{ Distributive law} \\
&\equiv ((\neg p \wedge \neg q) \vee (q \wedge \neg q)) \vee ((\neg p \wedge p) \vee (q \wedge p)), \text{ Distributive law (twice)}
\end{aligned}$$

$$\begin{aligned}
&\equiv ((\neg p \wedge \neg q) \vee F) \vee (F \vee (q \wedge p)), \text{ Negation law (twice)}\\
&\equiv (\neg p \wedge \neg q) \vee (q \wedge p), \text{ Identity law (twice)}\\
&\equiv (p \wedge q) \vee (\neg p \wedge \neg q), \text{ Commutative law (twice)}
\end{aligned}$$

3. (6 points) Predicates.

   (a) (2 points) Determine the truth value of the following statement, giving reasons. The domain is all real numbers. $\exists x(x^4 < x^2)$

   Solution: This is True. An example is $x = \frac{1}{10}$ for which $x^4 = \frac{1}{10000}$ and $x^2 = \frac{1}{100}$.

   (b) (2 points) Express using logical operators, quantifiers and predicates: "The negation of a contradiction is a tautology".

   Solution from the text: Let $T(x)$ mean that x is a tautulogy and $C(x)$ mean that x is a contradiction. Then $\forall x(C(x) \to T(\neg x)$.

   Note: The solution should also mention that the domain is all propositions. Since x is a proposition, $\neg x$ is well defined.

   (c) (2 points) Let $P(x), Q(x), R(x), S(x)$ be the statements "x is a baby", "x is logical", "x is able to manage a crocodile" and "x is despised" respectively. Suppose that the domain consists of all people. Express the following using quantifiers and the above predicates: "Nobody is despised who can manage a crocodile".

   Solution from the text: $\forall x(R(x) \to \neg S(x))$.

4. (6 points) Nested quantifiers

   (a) (2 points) Let $F(x, y)$ be the statement "x can fool y", where the domain consists of all people in the world. Use quantifiers to express the following statement: "Everyone can be fooled by somebody".

   Solution: $\forall x \exists F(y, x)$.

   (b) (2 points) Express the following statement in predicate logic: "Every real number has exactly 2 square roots".

   Solution from the text: $\forall x \exists a \exists b(a \neq b \wedge \forall c(c^2 = x \leftrightarrow (c = a \vee c = b)))$

   Note: There are other possible solutions.

   (c) (2 points) Express the negative of the following statement so that all negation symbols immediately precede predicates.

   $\forall x \exists y(P(x, y) \to Q(x, y))$

   Solution:

$$\begin{aligned}
\neg(\forall x \exists y(P(x,y) \to Q(x,y))) &\equiv \exists x \neg(\exists y(P(x,y) \to Q(x,y)))\\
&\equiv \exists x \forall y(\neg(P(x,y) \to Q(x,y)))\\
&\equiv \exists x \forall y(\neg(\neg P(x,y) \vee Q(x,y)))\\
&\equiv \exists x \forall y(P(x,y) \wedge \neg Q(x,y))
\end{aligned}$$

5. (6 points) Inference.

(a) (3 points) For the following premises, what relevant conclusion(s) can be drawn? Explain the rules of inference used to obtain each conclusion.
1. I am either dreaming or hallucinating.
2. I am not dreaming.
3. If I am hallucinating, I see elephants running down the road.

Solution: You can keep the sentences in English or convert them to propositional logic.

1.	I am not dreaming	Premise
2.	I am either dreaming or hallucinating.	Premise
3.	I am hallucinating	Disjunctive Syllogism from (2) and (1)
4.	If I am hallucinating, I see elephants running down the road.	Premise
5.	I see elephants running down the road.	Modus Ponens from (3) and (4)

(b) (3 points) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \to \neg S(x))$ and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

Solution from the text:

1.	$\exists x \neg P(x)$	Premise
2.	$\neg P(c)$	Existential instantiation from (1)
3.	$\forall x(P(x) \vee Q(x))$	Premise
4.	$P(c) \vee Q(c)$	Existential instantiation from (3)
5.	$Q(c)$	Disjunctive Syllogism from (2) and (4)
6.	$\forall x(\neg Q(x) \vee S(x))$	Premise
7.	$\neg Q(c) \vee S(c)$	Universal instantiation from (6)
8.	$S(c)$	Disjunctive Syllogism from (5) and (7)
9.	$\forall x(R(x) \to \neg S(x))$	Premise
10.	$R(c) \to \neg S(c)$	Universal instantiation from (9)
11.	$\neg R(c)$	Modus Tollens from (8) and (10)
12.	$\exists x \neg R(x)$	Existential generalization from (11)