In the following inference rules, H ranges over a finite set of sentences (formulae without free variables), ψ, ψ_1, ψ_2, ϕ, ϕ_1, ϕ_2 range over sentences, t, t_1, t_2 over terms without free variables, and x over variables of appropriate types.

1 Basic Rules

$$\frac{}{H, \phi \vdash \phi}\ \mathsf{Hyp} \qquad \frac{H \vdash \psi \quad H, \psi \vdash \phi}{H \vdash \phi}\ \mathsf{Cut} \qquad \frac{H \vdash \phi}{H, \psi \vdash \phi}\ \mathsf{Mon}$$

$$\frac{H, \psi_1 \vdash \phi \quad H, \psi_2 \vdash \phi}{H, \psi_1 \lor \psi_2 \vdash \phi}\ \mathsf{Or_L} \qquad \frac{H \vdash \phi_1}{H \vdash \phi_1 \lor \phi_2}\ \mathsf{Or_{R1}} \qquad \frac{H \vdash \phi_2}{H \vdash \phi_1 \lor \phi_2}\ \mathsf{Or_{R2}}$$

$$\frac{H, \psi_1, \psi_2 \vdash \phi}{H, \psi_1 \land \psi_2 \vdash \phi}\ \mathsf{And_L} \qquad \frac{H \vdash \phi_1 \quad H \vdash \phi_2}{H \vdash \phi_1 \land \phi_2}\ \mathsf{And_R}$$

$$\frac{H, \psi_1, \psi_2 \vdash \phi}{H, \psi_1, \psi_1 \Rightarrow \psi_2 \vdash \phi}\ \mathsf{Imp_L} \qquad \frac{H, \phi_1 \vdash \phi_2}{H \vdash \phi_1 \Rightarrow \phi_2}\ \mathsf{Imp_R}$$

$$\frac{H, \psi_1 \Rightarrow \psi_2, \psi_2 \Rightarrow \psi_1 \vdash \phi}{H, \psi_1 \Leftrightarrow \psi_2 \vdash \phi}\ \mathsf{Eqv_L} \qquad \frac{H, \phi_1 \vdash \phi_2 \quad H, \phi_2 \vdash \phi_1}{H \vdash \phi_1 \Leftrightarrow \phi_2}\ \mathsf{Eqv_R}$$

$$\frac{H \vdash \psi}{H, \neg\psi \vdash \phi}\ \mathsf{Not_L} \qquad \frac{H, \phi \vdash \bot}{H \vdash \neg\phi}\ \mathsf{Not_R} \qquad \frac{}{H \vdash \phi \lor \neg\phi}\ \mathsf{Classical}$$

$$\frac{}{H \vdash \top}\ \mathsf{True} \qquad \frac{}{H, \bot \vdash \phi}\ \mathsf{False}$$

$$\frac{H, \forall x \cdot \psi, \psi[x := t] \vdash \phi}{H, \forall x \cdot \psi \vdash \phi}\ \mathsf{All_L} \qquad \frac{H \vdash \phi[x := c]}{H \vdash \forall x \cdot \phi}\ \mathsf{All_R}$$

(c is a constant not occuring in H or ϕ.)

$$\frac{H \vdash \phi[x := t]}{H \vdash \exists x \cdot \phi}\ \mathsf{Ex_R} \qquad \frac{H, \psi[x := c] \vdash \phi}{H, \exists x \cdot \psi \vdash \phi}\ \mathsf{Ex_L}$$

(c is a constant not occuring in H, ψ or ϕ.)

$$\frac{H, t_1 = t_2 \vdash \phi}{H', t_1 = t_2 \vdash \phi'}\ \mathsf{Subst}$$

(H', ϕ' is obtained from H, ϕ, respectively, by replacing some occurrences of t_1 by t_2.)

$$\frac{}{\vdash t = t}\ \mathsf{Refl}$$

2 Derived Rules

$$\frac{H, \psi \vdash \phi \quad H, \neg\psi \vdash \phi}{H \vdash \phi}\ \mathsf{Case} \qquad \frac{H \vdash \psi_1 \quad H, \psi_1, \psi_2 \vdash \phi}{H, \psi_1 \Rightarrow \psi_2 \vdash \phi}\ \mathsf{MP}$$

$$\frac{H, \neg\phi \vdash \psi}{H, \neg\psi \vdash \phi}\ \mathsf{Not}'_{\mathsf{L}} \qquad \frac{H, \phi \vdash \neg\psi}{H, \psi \vdash \neg\phi}\ \mathsf{Not}'_{\mathsf{R}}$$

$$\frac{H, \neg\phi_2 \vdash \phi_1}{H \vdash \phi_1 \vee \phi_2}\ \mathsf{Or}'_{\mathsf{R1}} \qquad \frac{H, \neg\phi_1 \vdash \phi_2}{H \vdash \phi_1 \vee \phi_2}\ \mathsf{Or}'_{\mathsf{R2}}$$

$$\frac{H, \psi \vdash \phi}{H, \neg\neg\psi \vdash \phi}\ \mathsf{DNot}_{\mathsf{L}} \qquad \frac{H \vdash \phi}{H \vdash \neg\neg\phi}\ \mathsf{DNot}_{\mathsf{R}}$$