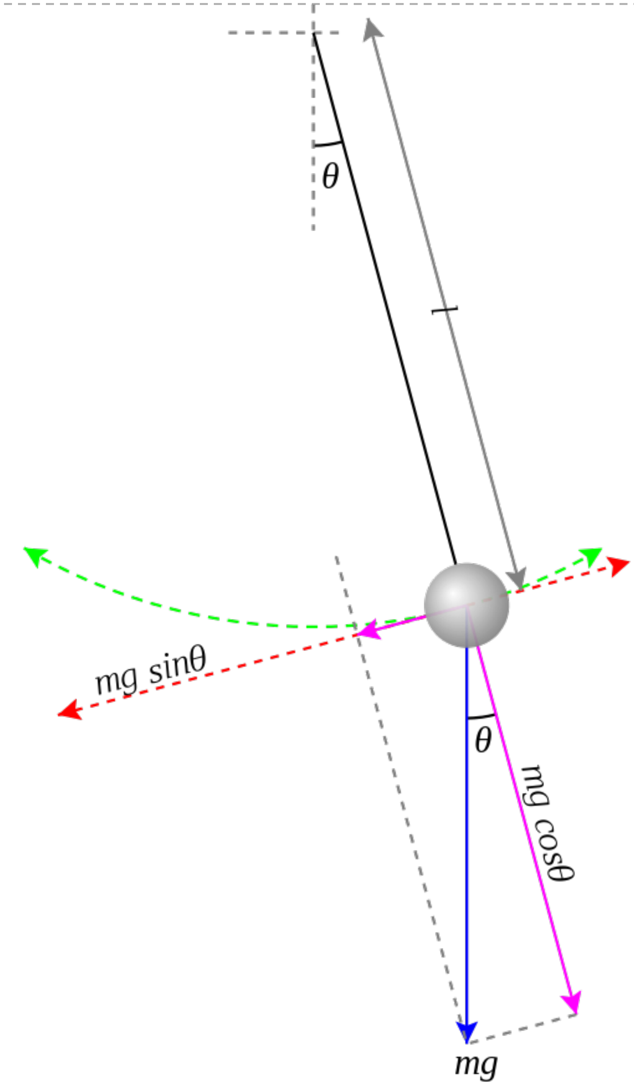
Ordinary differential equations

"Simple" pendulum

- the simple pendulum is a standard topic in most introductory physics courses
- the tangential component of the net force acting on the pendulum is

$$F = ma = -mg\sin\theta$$

$$\begin{aligned} a &= -g\sin\theta \\ &\approx -g\theta \end{aligned}$$



http://commons.wikimedia.org/wiki/File%3APendulum_gravity.svg

"Simple" pendulum

- the small angle approximation is required to find a simple solution for the horizontal position of the pendulum as a function of time

$$x(t) = x_0 \cos\left(\sqrt{\frac{g}{\ell}}\,t\right)$$

- using computational methods we can try to see what happens if we don't make the small angle approximation

"Simple" pendulum

- the differential equation describing the pendulum motion is normally written in terms of the angular position θ

$$a = -g \sin\theta$$

$$\alpha = \frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\sin\theta$$

"Simple" pendulum

- to use Euler's method, we use the same trick as for projectile motion
  - introduce the angular velocity ω

$$\frac{d\theta}{dt} = \omega$$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = -\frac{g}{\ell}\sin\theta$$

"Simple" pendulum

- we can now apply Euler's method

$$\frac{d\theta}{dt} \approx \frac{\theta_i - \theta_{i-1}}{\Delta t} = \omega_{i-1} \quad \Longrightarrow \quad \theta_i = \theta_{i-1} + \omega_{i-1}\Delta t$$

$$\frac{d\omega}{dt} \approx \frac{\omega_i - \omega_{i-1}}{\Delta t} = -\frac{g}{\ell}\sin\theta_{i-1} \quad \Longrightarrow \quad \omega_i = \omega_{i-1} - \frac{g}{\ell}\sin\theta_{i-1}\,\Delta t$$

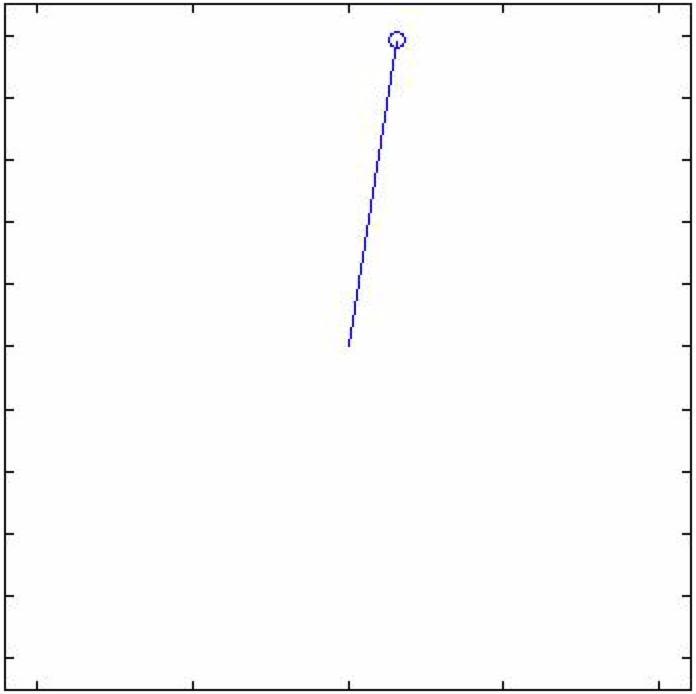
"Simple" pendulum

- the code is almost identical to that for projectile motion; only the changes are shown below:

```
% estimated angular position and velocity
theta = zeros(size(t));
omega = zeros(size(t));
theta(1) = theta0;
omega(1) = omega0;
for i = 2:n
    theta(i) = theta(i - 1) + omega(i - 1) * dt;
    omega(i) = omega(i - 1) - 9.81 / L * sin(theta(i - 1)) * dt;
end
```

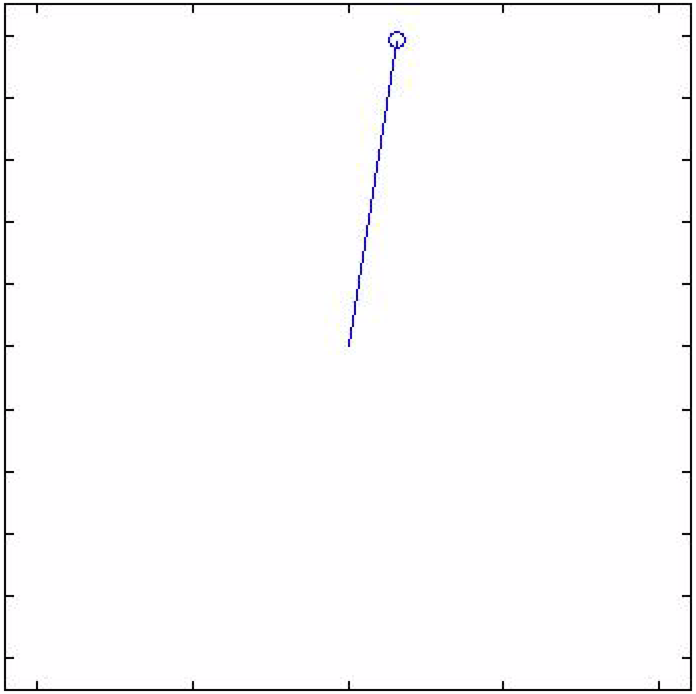
"Simple" pendulum

- (video clip) dt = 0.1



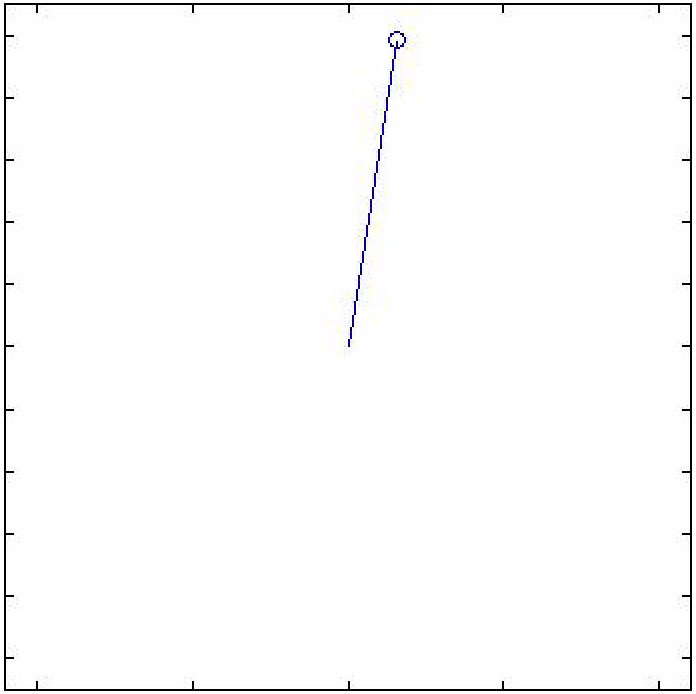
"Simple" pendulum

- (video clip) dt = 0.01



"Simple" pendulum

- (video clip) dt = 0.001



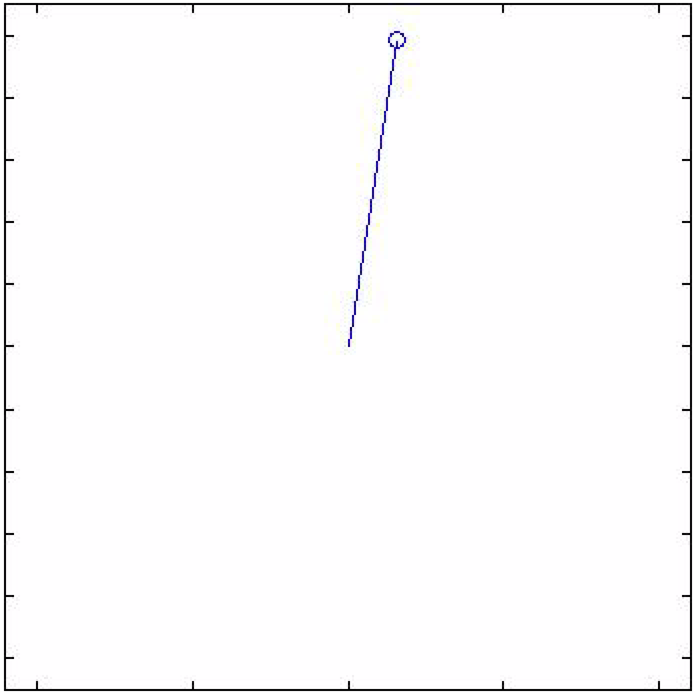
"Simple" pendulum

- it turns out that for any finite positive value of dt, Euler's method will cause the pendulum to swing faster and faster
  - it is possible to show that the energy of the pendulum continually increases using Euler's method

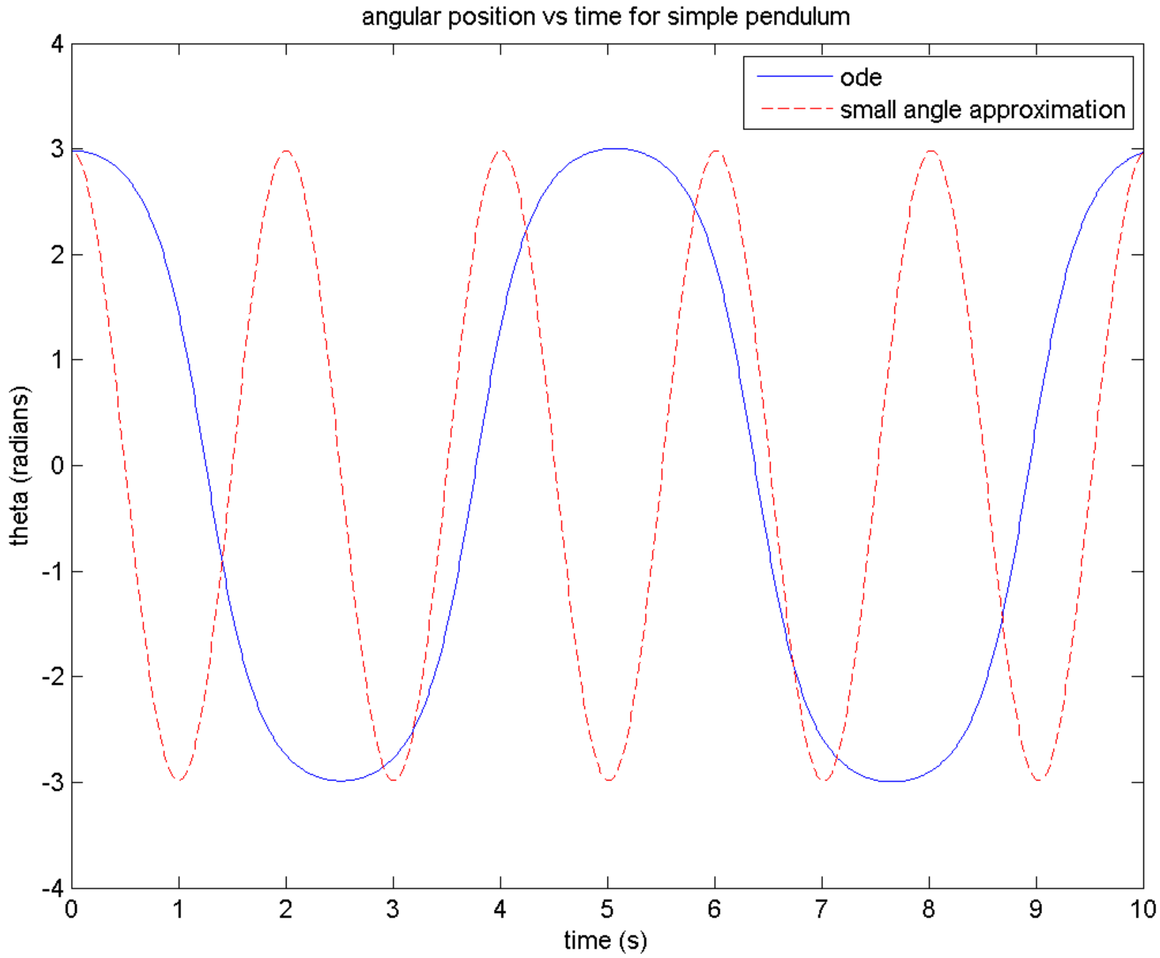
- this isn't completely surprising given the simplicity of Euler's method
- slightly more sophisticated methods are needed to obtain a more realistic solution

"Simple" pendulum

- (video clip) solved using MATLAB **ode45**



"Simple" pendulum



Summary

Weeks 1–6

- computer math is not the same as math
- MATLAB
  - creating and using vectors and matrices
  - indexing (numeric vs logical)
  - operators (normal form vs element-by-element form)
  - logical operators
  - if statements
  - loops
  - plotting

Week 7

- basic statistics
  - location
    - mean, median
  - spread
    - variance, standard deviation, interquartile range
  - percentiles
    - (requires the Statistics toolbox; you won't be asked to compute these)
  - boxplots

Week 8

- random numbers and simulation
  - rand, randi, randn, randperm
  - histograms

Week 9

- least squares line and polynomial fitting
  - polyfit, polyval
  - transforming non-linear curve fitting problems into linear curve fitting problems
  - principle of least squares and how the least squares problem can be solved mathematically

Week 10

- root finding
  - Newton's method
  - bisection method
    - the principle of recursion (but not how to implement a recursive method)
  - function functions
  - fzero

Week 11

- numerical differentiation
  - finite differences
- numerical integration
  - composite rectangle, trapezoid, and Simpson's rules
  - integral

Week 12

- ordinary differential equations
  - I do <u>not</u> expect you to be able to formulate a problem in terms of a differential equation
  - Euler's method