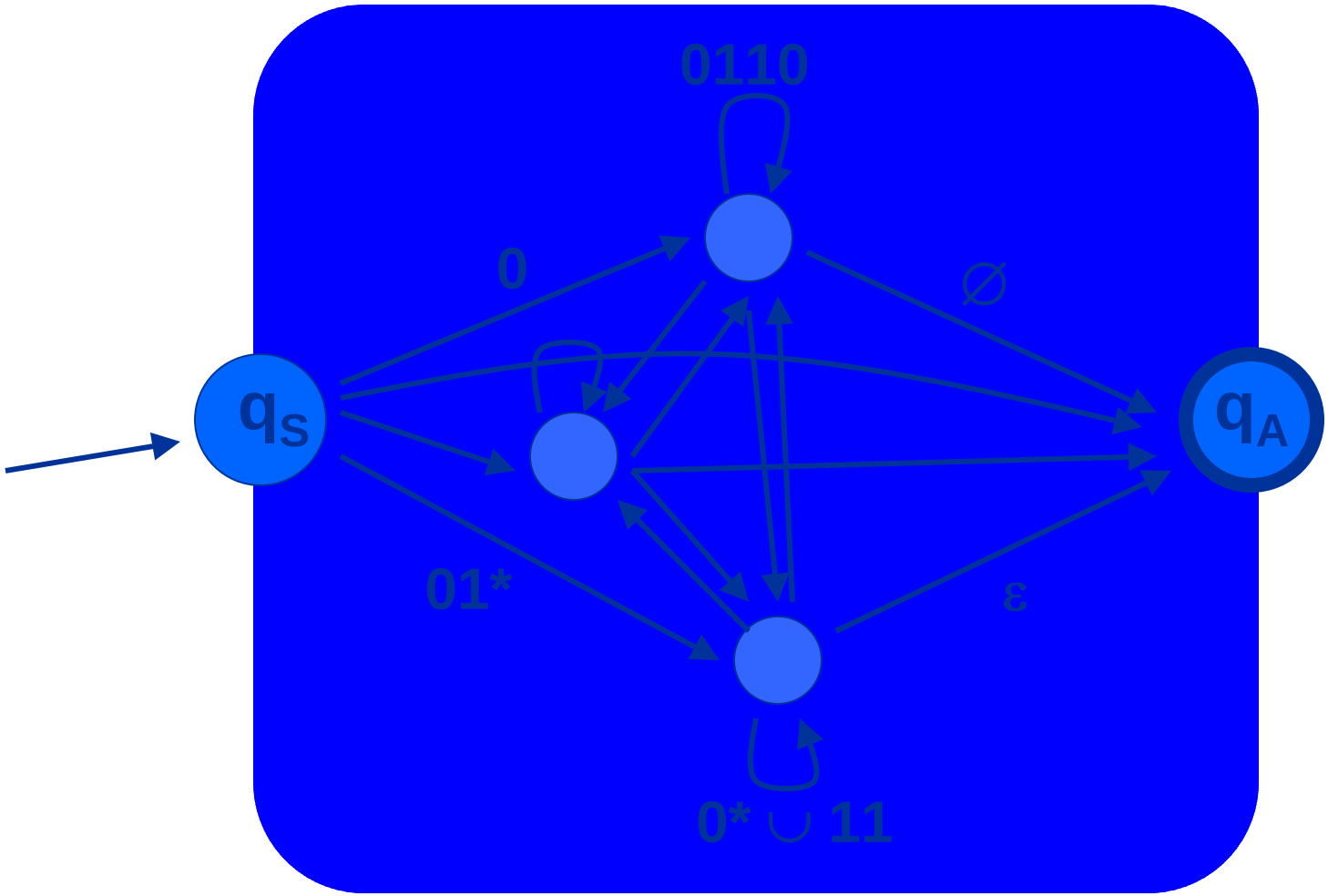
Back to RL ~ RE

- The second part (Lemma 1.60):
  If a language is regular, then it can be described by a regular expression.
- Proof strategy:
  - regular implies equivalent DFA.
  - convert DFA to GNFA (generalized NFA)
  - convert GNFA to NFA.

GNFA: NFA that have regular expressions as transition labels

Example GNFA



Generalized NFA - defn

Generalized non-deterministic finite automaton $M=(Q, \Sigma, \delta, q_{start}, q_{accept})$ with

- Q finite set of states
- Σ the input alphabet
- q_{start} the start state
- q_{accept} the (unique) accept state
- $\delta:(Q - \{q_{accept}\})\times(Q - \{q_{start}\}) \rightarrow \mathcal{R}$ is the transition function

($\mathcal{R}$ is the set of regular expressions over Σ)

(NOTE THE NEW DEFN OF δ)

Characteristics of GNFA's δ

- $\delta:(Q\backslash\{q_{accept}\})\times(Q\backslash\{q_{start}\}) \rightarrow \mathcal{R}$

The interior $Q\backslash\{q_{accept},q_{start}\}$ is fully connected by δ
From q_{start} only 'outgoing transitions'
To q_{accept} only 'ingoing transitions'
Impossible $q_i \rightarrow q_j$ transitions are labeled "$\delta(q_i,q_j) = \varnothing$"

Observation: This GNFA recognizes the language L(R)

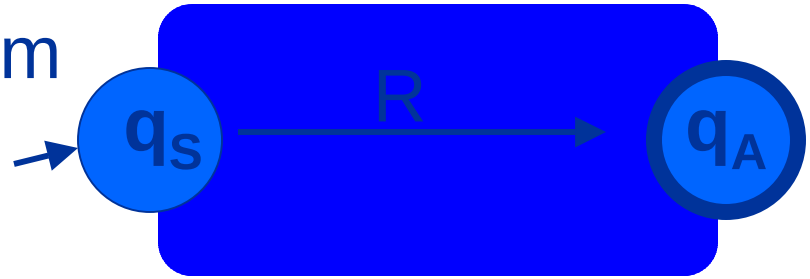
Proof Idea of Lemma 1.60

Proof idea (given a DFA M):

Construct an equivalent GNFA M' with $k \geq 2$ states

Reduce one-by-one the internal states until $k=2$

This GNFA will be of the form

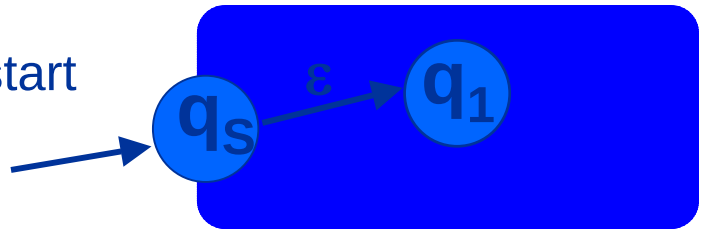


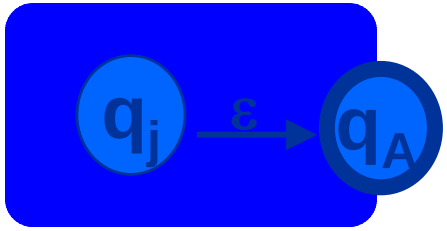
This regular expression R
will be such that L(R) = L(M)

DFA M → Equivalent GNFA M'

Let M have k states $Q=\{q_1,\ldots,q_k\}$

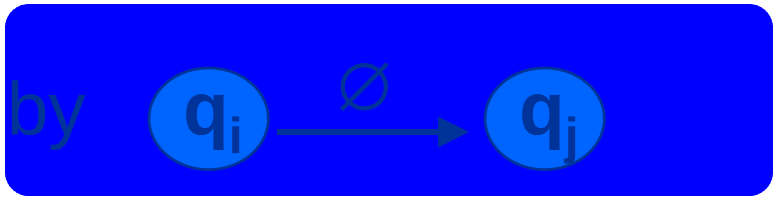
- Add two states q_{accept} and q_{start}
- Connect q_{start} to earlier q_1:



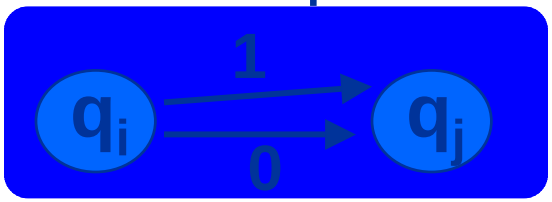


- Connect old accepting states to q_{accept}

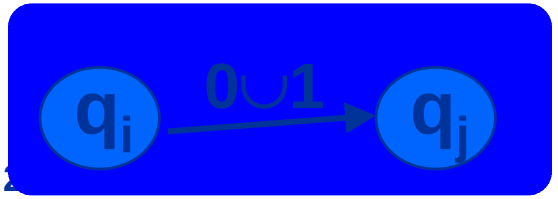
- Complete missing transitions by



- Join multiple transitions:



becomes



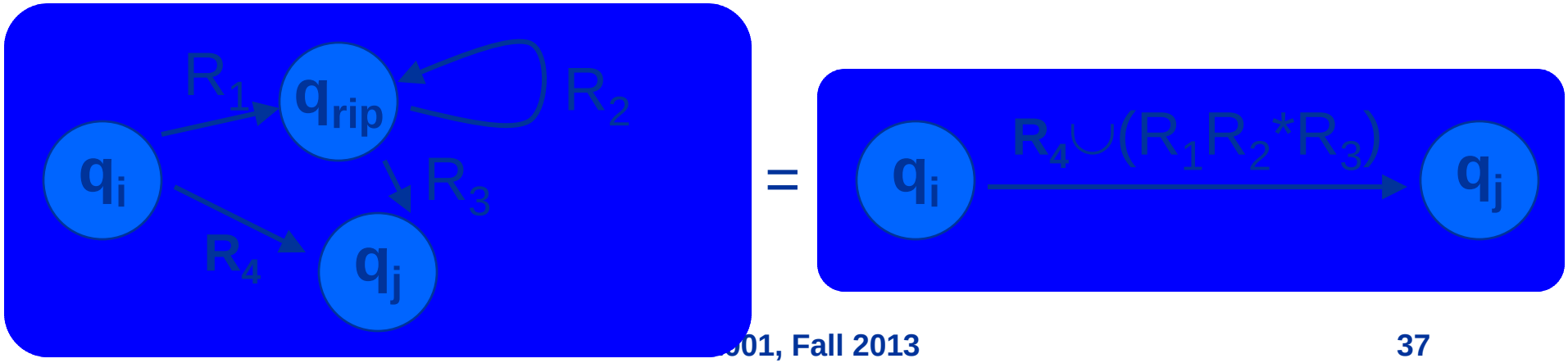
Remove Internal state of GNFA

If the GNFA M has more than 2 states, 'rip' internal q_{rip} to get equivalent GNFA M' by:

- Removing state q_{rip}: $Q' = Q \setminus \{q_{rip}\}$
- Changing the transition function δ by

$$\delta'(q_i,q_j) = \delta(q_i,q_j) \cup (\delta(q_i,q_{rip})(\delta(q_i,q_j))^*\delta(q_{rip},q_j))$$

for every $q_i \in Q' \setminus \{q_{accept}\}$ and $q_j \in Q' \setminus \{q_{start}\}$



Proof Lemma 1.60

Let M be DFA with k states

Create equivalent GNFA M’ with k+2 states

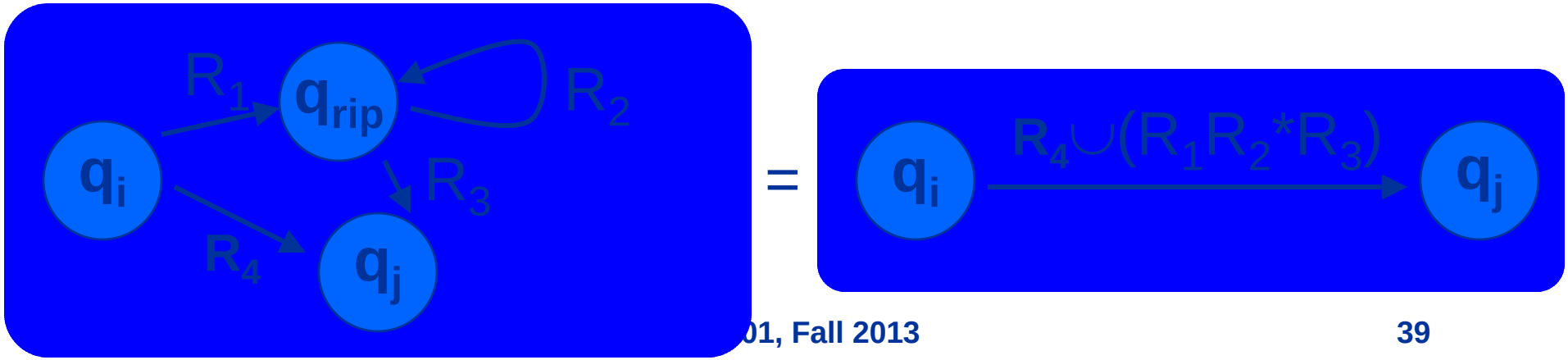
Reduce in k steps M’ to M’’ with 2 states

The resulting GNFA describes a single regular expressions R

The regular language L(M) equals the language L(R) of the regular expression R

Proof Lemma 1.60 - continued

- Use induction (on number of states of GNFA) to prove correctness of the conversion procedure.
- Base case: k=2.
- Inductive step: 2 cases – q_{rip} is/is not on accepting path.



Recap RL = RE

Let R be a regular expression, then there exists an NFA M such that L(R) = L(M)

The language L(M) of a DFA M is equivalent to a language L(M') of a GNFA = M', which can be converted to a two-state M''

The transition $q_{start} \xrightarrow{R} q_{accept}$ of M'' obeys L(R) = L(M'')

Hence: RE $\subseteq$ NFA = DFA $\subseteq$ GNFA $\subseteq$ RE

Example

L = {w| the sum of the bits of w is odd}