CSE 2001:
Introduction to Theory of Computation

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Recall: Regular Languages

The language recognized by a finite automaton M is denoted by **L(M)**.

A <u>regular language</u> is a language for which there exists a recognizing finite automaton.

Terminology: closure

- A set is defined to be closed under an operation if that operation on members of the set always produces a member of the same set. (adapted from wikipedia)

E.g.:

- The integers are closed under addition, multiplication.
- The integers are not closed under division
- Σ^* is closed under concatenation

- A set can be defined by closure -- Σ^* is called the (Kleene) closure of Σ under concatenation.

Terminology: Regular Operations

Pages 44-47 (Sipser, 3rd edition)

The regular operations are:

1. Union
2. Concatenation
3. Star (Kleene Closure): For a language A,

   $A^* = \{w_1 w_2 w_3 \ldots w_k \mid k \geq 0, \text{ and each } w_i \in A\}$

Closure Properties

- Set of regular languages is closed under
  -- Complementation
  – Union
  – Concatenation
  – Star (Kleene Closure)

Complement of a regular language

- Swap the accepting and non-accept states of M to get M'.

- The complement of a regular language is regular.

Other closure properties

Union: Can be done with DFA, but using a complicated construction.

Concatenation: We tried and failed

Star: ???

We introduced non-determinism in FA

Recall: NFA drawing conventions

- Not all transitions are labeled
- Unlabeled transitions are assumed to go to a reject state from which the automaton cannot escape

Closure under regular operations

Union (new proof):

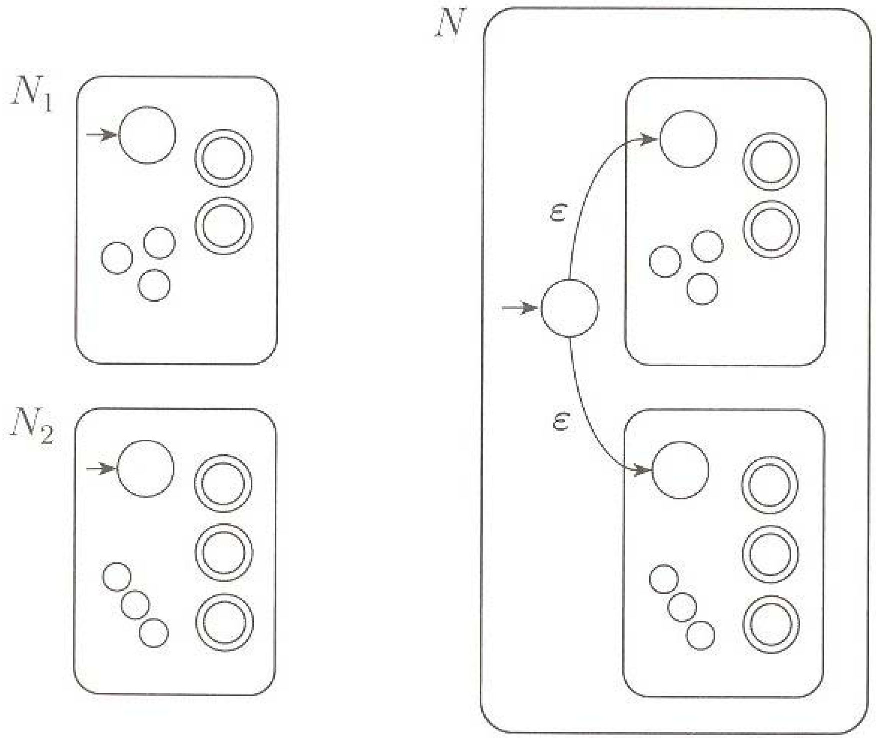

FIGURE 1.46
Construction of an NFA N to recognize $A_1 \cup A_2$

Closure under regular operations

Concatenation:

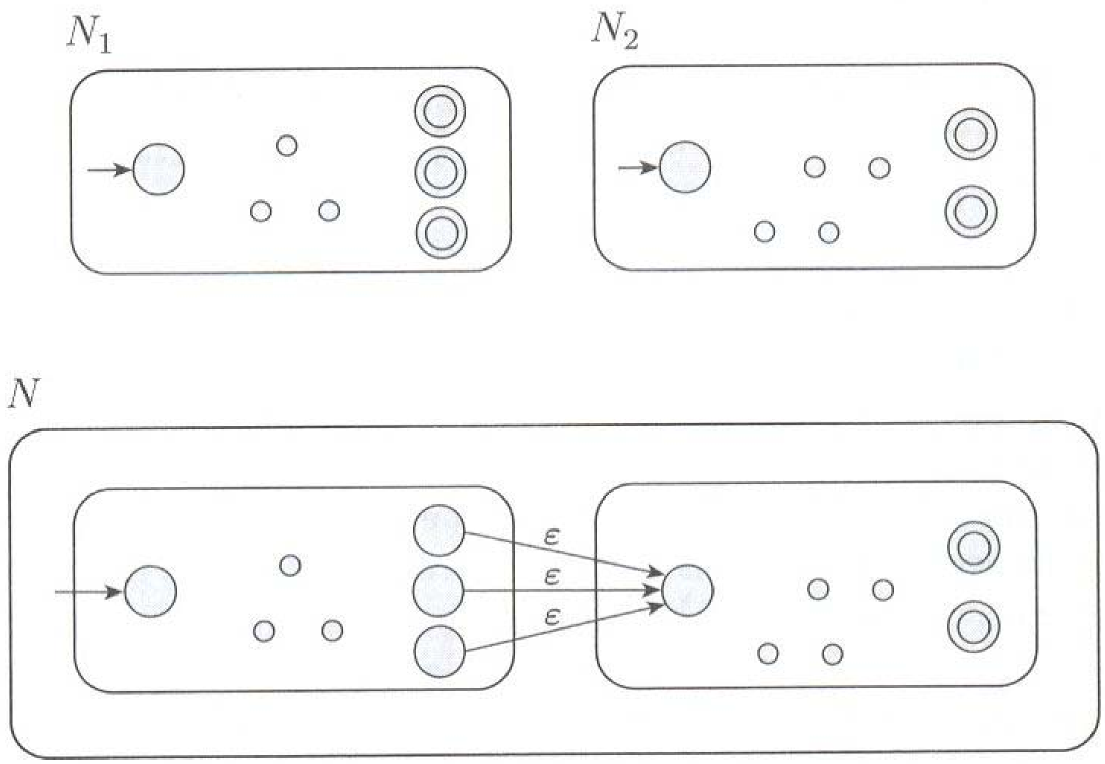

FIGURE 1.48
Construction of N to recognize $A_1 \circ A_2$

Closure under regular operations

Star:

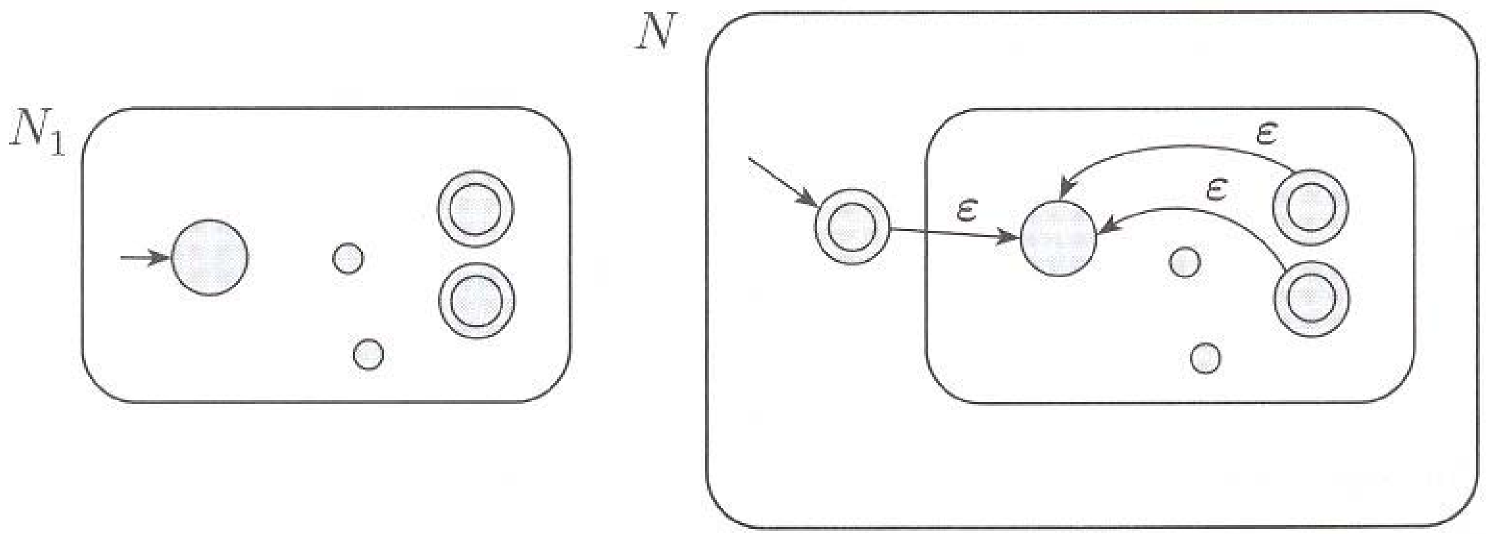

FIGURE 1.50
Construction of N to recognize A^*

Incorrect reasoning about RL

- Since $L_1 = \{w | w=a^n, n \in \mathbf{N}\}$, $L_2 = \{w | w = b^n, n \in \mathbf{N}\}$ are regular, therefore $L_1 \bullet L_2 = \{w | w=a^n b^n, n \in \mathbf{N}\}$ is regular
- If L_1 is a regular language, then $L_2 = \{w^R | w \in L_1\}$ is regular, and Therefore $L_1 \bullet L_2 = \{w w^R | w \in L_1\}$ is regular

Are NFA more powerful than DFA?

- NFA can solve every problem that DFA can (DFA are also NFA)
- Can DFA solve every problem that NFA can?