CSE 2001 Final examination
Fall 2012
Dec 6, 2012
Instructor: S. Datta

Name (LAST, FIRST): ______________________________

Student number: ______________________________

Instructions:

1. If you have not done so, put away all books, papers, cell phones and other electronic devices. Write your name and student number now!
2. Check that this examination has 8 pages. There should be 11 questions together worth 40 points.
3. You have 180 minutes to complete the exam. Use your time judiciously.
4. Show all your work. Partial credit is possible for an answer, but only if you show the intermediate steps in obtaining the answer.
5. If you need to make an assumption to answer a question, please state the assumption clearly.
6. Points will be deducted for **vague and ambiguous** answers.
7. Your answers MUST be LEGIBLE.

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

1. (3 points) Let $\Sigma = \{0, 1\}$. Design a DFA for the language of all words not ending with 10. In 1-2 sentences, argue that your design is correct.

2. (2 points) Construct a DFA or NFA that accepts the language corresponding to the regular expression $a^*(b \cup a^3)b^*$.

3. (6 points) Let $\Sigma = \{0, 1\}$. Let $\overline{w}$ be the string obtained from a binary string w by replacing every 0 with 1 and every 1 with 0.

   (a) (3 points) Prove that the following language is not regular.

$$L = \{w\overline{w} | w \in \Sigma^*\}$$

   (b) (3 points) Recall that the superscript R indicates string reversal. For the following language, indicate whether it is context-free. If you think it is, provide a CFG or a PDA for this language and a very short explanation of why it is correct. If you think it is not context-free, prove your answer.

$$L = \{w\overline{w}^R | w \in \{0, 1\}^*\}.$$

4. (3 points) Let $\Sigma = \{a, b\}$. Construct a CFG for the language $L = \{a^m b^n | m = 3n \text{ or } n = 3m, m, n \geq 0\}$.

5. (4 points) Let $\Sigma = \{0, 1\}$. Prove using induction that every string produced by the CFG with the following rules has more 0's than 1's.

$$S \rightarrow 0|S0|0S|1SS|SS1|S1S$$

6. (3 points) Construct a Turing Machine that takes 2 positive integers m, n (in normal base 10 form, so twenty three is represented as 23) and computes the function $f(m, n) = (m + 1)^n$.

7. (3 points) Describe a Turing machine that decides $A = \{0^{5^n} | n \geq 0\}$ – the language consisting of all strings of zeroes whose length is a power of 5.

8. (4 points) Without using Rice's Theorem prove the undecidability of the following language:

$$\text{INFINITE}_{TM} = \{< M > | M \text{ is a TM and } L(M) \text{ is an infinite language.}\}$$

9. (2+1 points) Describe a Turing Machine that accepts all binary strings with equal numbers of 0, 1's. Explain very briefly why this does not contradict the fact that the following language is undecidable.

$$L = \{< M > | M \text{ is a TM and every string accepted by } M \text{ has equal number of 0's and 1's}\}$$

10. (4 points) Without using Rice's Theorem prove the undecidability of the following language:

$$\text{NEQ}_{TM} = \{< M_1, M_2 > | M_1, M_2 \text{ are TM's and } L(M_1) \neq L(M_2)\}$$

11. (1+1+3 points) In one or two sentences, describe the significance of the Church-Turing thesis.
What is the difference between Turing-recognizable and Turing-decidable?
Show that the collection of Turing-recognizable languages is closed under the union operation.

Use this page if you need extra space