

List of useful formulae and facts.

1. $\log ab = \log a + \log b$

2. $\log a^n = n \log a$

3. $\log_b a = \log a / \log b$

4. $n = 2^{\lg n}$

5. $\frac{dx^k}{dx} = kx^{k-1}$

6.

$$\frac{d}{dx} \frac{u}{v} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

7.

$$\sum_{i=1}^n i = n(n+1)/2$$

8.

$$\sum_{i=1}^n i^2 = n(n+1)(2n+1)/6$$

9.

$$\sum_{i=1}^n i^3 = n^2(n+1)^2/4$$

10.

$$\sum_{i=0}^n x^i = \frac{x^{n+1} - 1}{x - 1}$$

11.

$$\sum_{i=0}^n x^i = \frac{1}{1-x}, \text{ when } |x| < 1.$$

12. For a monotonically increasing function $f(x)$,

$$\int_{m-1}^n f(x)dx \leq \sum_{k=m}^n f(k) \leq \int_m^{n+1} f(x)dx$$

13. Master Theorem : Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function, and let $T(n)$ be defined on the nonnegative integers by the recurrence $T(n) = aT(n/b) + f(n)$, where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$. Then $T(n)$ can be bounded asymptotically as follows.

(a) If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.

(b) If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \lg n)$.

(c) If $f(n) = \Omega(n^{\log_b a + \epsilon})$ for some constant $\epsilon > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n , then $T(n) = \Theta(f(n))$.