

CSE 3101, Summer 2013

Tutorial 2

May 13, 2013

1. For the following functions $f()$, $g()$, determine whether $f(n) = O(g(n))$ or $g(n) = O(f(n))$ or both.

- (a) $f(n) = \log n + \sqrt{n}$, $g(n) = n$.
- (b) $f(n) = \sqrt{n} \log n$, $g(n) = n$.
- (c) $f(n) = n + \sqrt{n}$, $g(n) = \log(n\sqrt{n})$.
- (d) $f(n) = 2^n$, $g(n) = 3^n$.

2. (Left as an exercise in Tutorial 1)

Analyze the running time of the following algorithm.

```
POWER( $y, z$ )
1  // return  $y^z$  where  $y \in R, z \in N$ 
2   $x \leftarrow 1$ 
3  while  $z > 0$ 
4  do if  $odd(z)$ 
5      then  $x \leftarrow x * y$ 
6       $z \leftarrow \lfloor z/2 \rfloor$ 
7       $y \leftarrow y^2$ 
8  return  $x$ 
```

3. What is the value returned by of the following program as a function of n ? Analyze its running time using $\Theta()$ notation.

```
FUNC1( $n$ )
1   $r \leftarrow 0$ 
2  for  $i \leftarrow 1$  to  $n - 1$ 
3  do for  $j \leftarrow i + 1$  to  $n$ 
4      do for  $k \leftarrow 1$  to  $j$ 
5          do  $r \leftarrow r + 1$ 
6  return  $r$ 
```

4. What is the value returned by of the following program as a function of n ? Analyze its running time using $\Theta()$ notation.

```
FUNC2( $n$ )
1   $r \leftarrow 0$ 
2  for  $i \leftarrow 1$  to  $n$ 
3  do for  $j \leftarrow 1$  to  $i$ 
4      do for  $k \leftarrow j$  to  $i + j$ 
5          do  $r \leftarrow r + 1$ 
6  return  $r$ 
```

5. Prove by induction on n that for $n \geq 1$,

$$\sum_{i=1}^n \frac{1}{i^2} < 2 - \frac{1}{n}$$

6. Prove by induction on n that for $n \geq 0$, $n^5 - n$ is divisible by 5.
7. Can you prove that the following algorithm for finding the maximum value of an array $S[1..n]$ is correct?

```
MAXIMUM( $x, y$ )
1  Comment: return maximum value in  $S[x..y]$ 
2  if  $y - x \leq 1$ 
3      then return  $\max(S[x], S[y])$ 
4      else  $max1 \leftarrow \text{MAXIMUM}(x, \lfloor (x + y)/2 \rfloor)$ 
5            $max2 \leftarrow \text{MAXIMUM}(\lfloor (x + y)/2 \rfloor + 1, y)$ 
6  return  $\max(max1, max2)$ 
```