CSE 3101: Design and Analysis of Algorithms

Assignment 4, Weight: 4%,
Due: July 9, in the drop box by 6:45 pm or in class by 7:10 pm

1. You are considering renting a moving truck to help you move from Montreal to Toronto. You want to optimize the amount of gas required.

   The truck guzzles a liter of gas for every km it travels. It has a 200-liter gas tank. When you rent the truck in Montreal, the tank is half-full. When you return it in Toronto, the tank must be at least half-full or you will really get gouged by the rental company. You would like to spend as little as possible for gas but you do not want to run out along the way.

   You are given the distance d in kilometers from Montreal to Toronto and the distance from Montreal of all gas stations along the way as well as the gas price at each station. Your algorithm should compute the amount you need to spend on gas. Prove optimal substructure and then provide a dynamic programming algorithm for this problem.

   NOTE: Assume that all numbers in this problem are integers (so gas prices are in whole cents). Also, a solution may not exist for certain instances of the problem. In such cases the algorithm should output "No solution".

2. *Arbitrage* is the use of discrepancies in currency exchange rates to make a profit. E.g., there may be a window of time during which 1 US dollar buys 0.75 British pounds, 1 British pound buys 2 Australian dollars, and 1 Australian dollar buys 0.70 US dollars. Then, a smarter trader can trade 1 US dollar and end up with $0.75 \times 2 \times 0.7 = 1.05$ US dollars, a profit of 5%. Suppose that there are n currencies $c_1, \ldots, c_n$ and a $n \times n$ table R of exchange rates, so that one unit of currency c_i buys $R[i, j]$ units of currency c_j. Design and analyze a dynamic programming algorithm to determine the maximum profit that can be made in exactly k currency conversions starting from currency c_1.