CSE 3101: Design and Analysis of Algorithms – Assignment 1
Weight: 2%, Due: May 14, in the drop box by 6:45 pm or in class by 7:10 pm

1 Solved Problems

1. Prove that $(n+1)^3 \in \Theta(n^3)$.

   Solution: Since

   $$\begin{aligned}(n+1)^3 &= n^3 + 3n^2 + 3n + 1 \\ &\leq n^3 + 3n^3 + 3n^3 + n^3 \\ &= 8n^3 \text{ for } n > 0\end{aligned}$$

   it follows that $(n+1)^3 \in O(n^3)$.

   Similarly since $(n+1)^3 > n^3$ for $n > 0$, it follows that $(n+1)^3 \in \Omega(n^3)$.

   Therefore $(n+1)^3 \in \Theta(n^3)$.

2. Find the relationship between $f(n) = n2^n$ and $g(n) = 3^n$ in terms of $O(), \Theta(), o(), \Omega(), \omega()$ and prove your answer.

   Solution: We will prove that $f(n) = o(g(n))$. As usual, the limit definition is the easiest to use.

   Since

   $$\begin{aligned}\lim_{n\to\infty} \frac{f(n)}{g(n)} &= \lim_{n\to\infty} \frac{n}{1.5^n} \\ &= \lim_{n\to\infty} \frac{1}{1.5^n \log_e 1.5} \\ &= 0\end{aligned}$$

   the result follows.

2 Problems to submit

1. Prove that $3n^2 + \sqrt{n} \in \Theta(n^2)$.
2. What is the value returned by the following function? Express your answer as a function of n. Give using $O()$ notation the worst-case running time.

```
EVAL1(n)
1  v ← 0
2  for i ← 1 to n
3  do for j ← i + 1 to i + n
4       do v ← v + 1
5  return v
```