Dept. of Computer Science & Engineering
York University

Assignment 4

Total marks: 50.

Out: July 27
Due: August 2 at 17:00 in the dropbox

Note that:

- The assignment can be handwritten or typed. It **must** be legible.
- You must do this assignment individually.
- Submit this assignment only if you have read and understood the policy on academic honesty on the course web page. If you have questions or concerns, please contact the instructor.
- Use the dropbox near the main office to submit your assignments, or hand them in at the beginning of class (please note the times and day above). No late submissions will be accepted.

1. [10 points] A Turing machine with a doubly infinite tape is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of Turing machine recognizes the class of Turing-recognizable languages.

2. [10 points] Let $\mathcal{N}$ be the set of all natural numbers.

   a) Prove that the set of all *finite* subsets of $\mathcal{N}$ is countable.

   b) Prove that the set of all subsets of $\mathcal{N}$ (i.e. $\mathcal{P}(\mathcal{N})$) is uncountable.

3. [20 points]
   Let $EQ_{TM} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are Turing machines and } L(M_1) = L(M_2)\}$.
   Let $SUB_{TM} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are Turing machines and } L(M_1) \subseteq L(M_2)\}$.

   a) If you had an algorithm $A(\langle M_1, M_2\rangle)$ that decided SUB_{TM}, show how you could use A as a subroutine to design an algorithm $B(\langle M_1, M_2\rangle)$ that decides EQ_{TM}.

   b) Use Theorem 5.4 of the Sipser textbook to prove that SUB_{TM} is undecidable.

   c) Assume that there is an algorithm $E(\langle M_1, M_2\rangle)$ that recognizes SUB_{TM}.
   Consider the following algorithm F:

$F(\langle M, w\rangle)$ % Here M is a Turing machine and w is an input string for M
Construct a Turing machine M_2 that accepts every string over M's alphabet *except* w
Run $E(\langle M, M_2\rangle)$ and output whatever E outputs
end F

Show that $F(\langle M, w\rangle)$ outputs "accept" if and only if M does not accept w.

d) Is SUB_{TM} Turing-recognizable? Prove that your answer is correct.

4. [10 points] Let $INTERSECT_{TM} = \{\langle M_1, M_2\rangle \mid M_1$ and M_2 are Turing machines and $L(M_1) \cap L(M_2) \neq \{\}\}$. Give a deterministic algorithm that recognizes $INTERSECT_{TM}$. Explain why your solution is correct.