

### Assignment 3

Total marks: 60.

*Out:* July 15

*Due:* July 25 by 18:00 in the dropbox.

Note that:

- The assignment can be handwritten or typed. It **must** be legible.
  - You must do this assignment individually.
  - Submit this assignment only if you have read and understood the policy on academic honesty on the course web page. If you have questions or concerns, please contact the instructor.
  - Use the dropbox near the main office to submit your assignments, or hand them in at the beginning of class (please note the times and day above). No late submissions will be accepted.
1. [15 points] Let  $L_1 = \{0^n 1^m \mid 0 < n \leq m < 2n\}$ . Let  $G_1$  be the grammar with starting symbol  $S$  and the following rules:

Rule 1:  $S \rightarrow 0S11$

Rule 2:  $S \rightarrow T$

Rule 3:  $T \rightarrow 0T1$

Rule 4:  $T \rightarrow 01$

The goal of this question is to prove that  $G_1$  generates the language  $L_1$ .

- a)** For each  $n$  and  $m$  satisfying  $0 < n \leq m < 2n$ , describe a leftmost derivation of  $0^n 1^m$  using the grammar  $G_1$ . (That is, say how many times to apply each rule and in what order.)
- b)** Give a formal proof that every string that can be generated from  $T$  is of the form  $0^k 1^k$  for some positive integer  $k$ .
- c)** Give a formal proof that every string generated by  $G_1$  is in  $L_1$ .
2. [10 points] Let  $L_2 = \{a^i b^j c^k \mid i, j, k \geq 0, i = j \text{ or } j = k\}$ . Describe a pushdown automaton that recognizes  $L_2$  by giving its state diagram.
3. [10 points] Let  $L_3 = \{a^n b^m c^n \mid n \geq m\}$ . Prove that  $L_3$  is not a context free language.

4. [15 points] We have seen in class that  $L = \{ww \mid w \in \{0,1\}^*\}$  is not a context free language. However the complement of  $L$ ,  $\bar{L} = (\{0,1\}^* \setminus L)$  is a context free language. Give a context free grammar  $G$  that generates  $\bar{L}$  and explain its design (you need not give a formal proof that it generates  $\bar{L}$ ).
5. [10 points] Let  $L_5 = \{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$ . Describe a Turing Machine that decides  $L_5$ . While you need not draw a state diagram, you should describe the machine in detail.