

## Assignment 2

Total marks: 90.

*Out:* June 24

*Due:* July 8 by 18:45 in the dropbox, or 19:00 in class

Note that:

- The assignment can be handwritten or typed. It **must** be legible.
  - You must do this assignment individually.
  - Submit this assignment only if you have read and understood the policy on academic honesty on the course web page. If you have questions or concerns, please contact the instructor.
  - Use the dropbox near the main office to submit your assignments, or hand them in at the beginning of class (please note the times and day above). No late submissions will be accepted.
1. [20 points] If  $L_1$  and  $L_2$  are two languages over the alphabet  $\Sigma$ , then we define  $L_1 \diamond L_2$  to be the language  $\{x_1y_1x_2y_2 \dots x_ny_n \mid n \geq 0, \text{ each } x_i \text{ and } y_i \text{ are in } \Sigma^*, x_1x_2 \dots x_n \in L_1 \text{ and } y_1y_2 \dots y_n \in L_2\}$ . That is, each string in the language  $L_1 \diamond L_2$  is formed by interleaving one string from  $L_1$  and one string from  $L_2$ .
    - a) If  $L_1 = \{blue, red\}$  and  $L_2 = \{010\}$ , write down three strings that are in  $L_1 \diamond L_2$ .
    - b) If  $L_1 = \{1\}^*$  and  $L_2 = \{010\}$ , write down a regular expression that describes  $L_1 \diamond L_2$ .
    - c) Prove that for all regular languages  $L_1$  and  $L_2$ ,  $L_1 \diamond L_2$  is also regular. If your proof involves constructing an automaton for  $L_1 \diamond L_2$ , give a formal specification of this automaton. Also say which strings take your automaton to each one of its states (you don't have to prove this).
  2. [10 points] Prove that for any regular expressions  $\alpha$  and  $\beta$ ,  $(\alpha \cup \beta)^* = (\alpha^* \circ \beta^*)^*$ .
  3. [10 points] Prove that  $\{a^n b^{2n} \mid n \geq 0\}$  is not regular.
  4. [10 points] Prove that  $\{a^n b a^m b a^{n+m} \mid n, m \geq 0\}$  is not regular.

5. [20 points] Let  $L_1 = \{a^i b^j c^k \mid i, j, k \geq 0, i = j \text{ or } j = k\}$ .
- a) Give a context free grammar for  $L_1$ . You don't have to prove that your answer is correct.
  - b) Show that your grammar is ambiguous.
6. [20 points] Consider the following CFG  $G$  over the alphabet  $\{a, b\}$ :

$$\begin{aligned} S &\rightarrow aB \mid bA \\ A &\rightarrow a \mid aS \mid BAA \\ B &\rightarrow b \mid bS \mid ABB \end{aligned}$$

- a) Show that  $ababba \in L(G)$ .
- b) Prove that  $L(G)$  is the set of all non-empty strings over the alphabet  $\{a, b\}$  that have an equal number of  $a$ 's and  $b$ 's.