

Assignment 1

Total marks: 90.

Out: May 20

Due: June 3 by 18:45 in the dropbox, or 19:00 in class

Note that:

- The assignment can be handwritten or typed. It **must** be legible.
- You must do this assignment individually.
- Submit this assignment only if you have read and understood the policy on academic honesty on the course web page. If you have questions or concerns, please contact the instructor.
- Use the dropbox near the main office to submit your assignments, or hand them in at the beginning of class (please note the times and day above). No late submissions will be accepted.

1. [15 points] Write each of the following sets explicitly:

- a) $\{a\} \times \{a, b\} \times \{a, b, c\}$
- b) $\emptyset \times \{a, b, c\}$
- c) $\mathcal{P}(\{a, b\}) \times \{a, b\}$

2. [10 points] Give a recursive definition for each of the following sets:

- a) $\{m \mid m = 5k + 1, k \in \mathcal{N}\}$
- b) $\{a^{3n} \mid n \in \mathcal{N}\}$

3. [15 points] Let the predicate $S(x)$ represent the statement “ x is a student”, $C(y)$ represent “ y is a course”, and $L(x, y)$ represent “ x likes y ”. Write sentences in predicate logic that represent the following English statements:

- a) There is a course that no student likes.
- b) Every student likes some course.
- c) Some student dislikes two distinct courses.

4. [10 points] Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
5. [10 points] Prove by induction on $|A|$ that for any finite set A , $|\mathcal{P}(A)| = 2^{|A|}$.
6. [10 points] Let M be a deterministic finite automaton. Under exactly what circumstances is $\epsilon \in L(M)$ (where ϵ denotes the empty string)? Prove your answer.
7. [20 points] Define formally deterministic finite automata accepting each of the following languages. Also provide an English description of what each state represents.
 - a) $\{w \in \{c, d\}^* \mid w \text{ has } dccd \text{ as a substring}\}$
 - b) $\{w \in \{c, d\}^* \mid w \text{ has an odd number of } c\text{'s and an even number of } d\text{'s}\}$