

Homework Assignment #6

Due: November 23, 2012

1. The integer programming (IP) problem is defined as follows: Given an $m \times n$ matrix A of integers and a vector $b \in \mathbb{Z}^m$, does there exist a vector $x \in \mathbb{Z}^n$ such that $Ax \leq b$?

The 2-IP problem is a special case where each row of A contains at most 2 non-zero values. Prove that 2-IP is **NL**-hard.

2. If L is a language over the alphabet $\{0, 1\}$, define $L' = \{x\#0^{|x|^2} : x \in L\}$.

(a) Prove that $L' \in \mathbf{P} \Rightarrow L \in \mathbf{P}$.

(b) Prove that there is a language L such that $L' \in \mathbf{SPACE}(\sqrt{n})$ and $L \notin \mathbf{SPACE}(\sqrt{n})$.
Hint: use the space hierarchy theorem.

(c) Prove that $\mathbf{P} \neq \mathbf{SPACE}(\sqrt{n})$.