Homework Assignment #4
Due: October 17, 2012

1. Suppose you have a collection of tasks. Each task takes one hour to complete. You can only work on one task at a time. Each task has a specified set of prerequisite tasks: before performing a task, you must complete at least one of its prerequisites. Each task also has a deadline. You want to know if it is possible to complete at least k of the tasks prior to their deadlines.

Formally, the problem is defined as follows.

Input: n tasks with deadlines $d_1, d_2, \ldots, d_n \in \mathbb{N}$ and prerequisite sets $P_1, P_2, \ldots P_n$, where $P_i \subseteq \{1, 2, \ldots, n\}$, and $k \in \mathbb{N}$.

Question: Does there exist a permutation $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ such that

- for all i, if $P_i \neq \{\}$ then there is a $j \in P_i$ such that $\sigma(j) < \sigma(i)$, and
- $|\{i : \sigma(i) \leq d_i\}| \geq k$?

(Here, $\sigma(i)$ represents the time slot assigned to job i.)

Prove that this problem is NP-complete. Try to make your proof as simple as possible.

2. Suppose an experiment provides you with a collection of data points. You want to group the data points so that points in the same group are close together. Formally, we define the problem as follows.

Input: A collection of n data points with pairwise distances $d_{i,j} \in \mathbb{N}$ for $1 \leq i < j \leq n$, and natural numbers K and W.

Question: Is there a way to partition $\{1, 2, \ldots, n\}$ into K sets $A_1, \ldots, A_K$ such that for all x and for all $i, j \in A_x$, $d_{i,j} < W$?

Prove this problem is NP-complete.

Hint: Your answer can be very short and simple if you choose the right problem to reduce.