Homework Assignment #3
Due: October 10, 2012

1. A no-writing Turing machine is one that never changes any character written on its tape. (Thus, in the transition function of such a machine, if $\delta(q, a) = (q', a', d)$ then $a = a'$.) Let L be a language. Assume there is a no-writing Turing machine M that decides L.

(a) Prove that there exists a constant k such that, for every input string x, M never visits the same square of the tape more than k times.

(b) Show that you can construct a no-writing Turing machine M' that decides L without ever moving beyond the first $n + 2$ squares of the tape on any input of length n.

(c) Prove that $L \in TIME(n)$.

2. A Yes-No-Maybe Turing machine (YNMTM) is a nondeterministic Turing machine that has three possible output states, $q_{YES}, q_{NO}, q_{MAYBE}$. The machine halts when it reaches any one of these states. The running time of a YNMTM on input string x is measured the same way as for an ordinary non-deterministic Turing machine: it is the maximum number of steps the machine can take before halting when it is given string x.

We say that a YNMTM M decides language L if

- There is no string x such that two computations of M on input x lead to q_{YES} and q_{NO}, respectively.
- For every string $x \in L$, some computation of M on input x leads to q_{YES}.
- For every string $x \notin L$, some computation of M on input x leads to q_{NO}.

Intuitively, this means that M never gives a false answer, and for every string, M is capable of giving the correct answer.

Prove that $L \in NP \cap co\text{-}NP$ iff there is a polynomial time YNMTM that decides L.