

Homework Assignment #8

Due: December 3, 2012

1.

- (a) Show that every **PSPACE**-hard problem is also **NP**-hard.
- (b) Show that if every **NP**-hard problem is also **PSPACE**-hard, then **PSPACE** = **NP**.

2. Recall the notion of a context-free grammar (CFG). It is defined by

- a finite alphabet V of variables,
- a finite alphabet T of terminals,
- a start symbol $S \in V$, and
- a finite set of rules R , where each rule is of the form $A \rightarrow \alpha$ with $A \in V$ and $\alpha \in (V \cup T)^*$.

We say that a string $x \in T^*$ is generated by the grammar if there is a sequence $\beta_0, \beta_1, \beta_2, \dots, \beta_k$ such that

- for all i , $\beta_i \in (V \cup T)^*$,
- $\beta_0 = S$,
- $\beta_k = x$, and
- for all i , there exists a rule $A \rightarrow \alpha$ such that β_{i+1} can be obtained from β_i by replacing one occurrence of A in β_i by α .

(Notice that strings generated by the grammar contain only terminals and no variables.)

Prove that the problem of determining whether a given CFG generates at least one string is **P**-complete.

Hint: use a reduction involving the monotone circuit value problem. Each variable of the CFG will correspond to a node of the circuit.