Homework Assignment #5
Due: November 5, 2012

1. The NICE problem is defined as follows.
Input: an undirected graph $G = (V, E)$, number $k \in \mathbb{N}$.
Question: Is there a nice set of k vertices $S \subseteq V$, where nice means that every vertex in $V - S$ is connected to at least one vertex in S by an edge?

We wish to show that NICE is NP-hard by reducing VERTEX-COVER to it. Let $f(V, E, k) = (V', E', k')$ where
$V' = V \cup \{v_e : e \in E\}$ (this is a disjoint union),
$E' = E \cup \{\{v, v_e\} : v \in V, e \in E$ and v is an endpoint of e in $G\}$, and
$k' =$??? (you must fill this in).
(The construction of V' and E' corresponds to adding to the original graph, for each edge in E, a new vertex connected to both of the edge's endpoints. Another way of looking at it is replacing each edge $e \in E$ by a little triangle consisting of the two endpoints of e and a new vertex v_e.)

(a) Prove that if there is a nice set $S \subseteq V'$ for $G' = (V', E')$ then there is a nice set $T \subseteq V$ for G' with $|T| \leq |S|$.

(b) Give a value for k' so that f is a reduction from VERTEX-COVER to NICE. Hint: the input graph need not be connected.

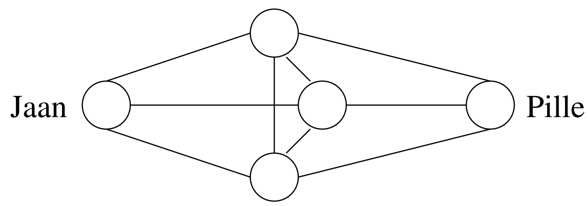
(c) Prove that f is a reduction from VERTEX-COVER to NICE (using the value of k' you defined in (b)).

2. Suppose there are n children who must be divided into two teams, team 0 and team 1. Some pairs of children know each other. If any *three* children who all know each other get placed on the same team, they will waste too much time chatting. Is it possible to split the children into teams so that no team has three mutual friends?

More formally, the CHATTY-CHILDREN (CC) problem can be formalized as follows.
Input: An undirected graph $G = (V, E)$.
Question: Is there a subset $S \subseteq V$ such that S contains no 3-clique and $V - S$ contains no 3-clique?

The goal of this problem is to show that $\neq$SAT (which is defined and shown to be NP-complete in Exercise 7.26 of the textbook; Exercise 7.24 of the second edition) can be reduced to CC.

(a) Show that in the following friendship graph G_1 (called a triangular bipyramid) it is possible to split the children into teams, but Jaan and Pille must be placed on the same team.



(b) Design a small friendship graph G_2, such that it is possible to split the children into teams, but two particular children, Tiiu and Kaja, must be placed on opposite teams. Explain why your answer is correct.

(c) Show that $\neq$SAT $\leq_P$ CC.