Homework Assignment #4
Due: October 17, 2012

1. Recall that an input for VERTEX-COVER or for CLIQUE consists of a vertex set V, an edge set E and a number $k \in \mathbb{N}$.

The textbook proves that VERTEX-COVER is NP-hard by reducing 3SAT to it. A different way to prove this would be to show that CLIQUE $\leq_p$ VERTEX-COVER, since we already know CLIQUE is NP-hard. This could be done using the reduction

$$f(V, E, k) = (V, \overline{E}, |V| - k),$$

where $\overline{E}$ denotes the complement of E. Write a careful proof that this reduction is correct, i.e., that $(V, E, k) \in$ CLIQUE $\Leftrightarrow f(V, E, k) \in$ VERTEX-COVER.

2. Suppose you have a collection of tasks. Each task takes one hour to complete. You can only work on one task at a time. Each task has a specified set of prerequisite tasks: before performing a task, you must complete at least one of its prerequisites. Each task also has a deadline. You want to know if it is possible to complete at least k of the tasks prior to their deadlines.

Formally, the problem is defined as follows.
Input: n tasks with deadlines $d_1, d_2, \ldots, d_n \in \mathbb{N}$ and prerequisite sets $P_1, P_2, \ldots P_n$, where $P_i \subseteq \{1, 2, \ldots, n\}$, and $k \in \mathbb{N}$.
Question: Does there exist a permutation $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ such that

- for all i, if $P_i \neq \{\}$ then there is a $j \in P_i$ such that $\sigma(j) < \sigma(i)$, and
- $|\{i : \sigma(i) \leq d_i\}| \geq k$?

(Here, $\sigma(i)$ represents the time slot assigned to job i.)

Prove that this problem is NP-complete. Try to make your proof as simple as possible.